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MODELING AN INTERFEROMETRIC FIBER OPTIC GYROSCOPE

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FOREWORD

This report presents the computations involved in the development of a general Interferometric Fiber Optic Gyroscope model. The work was done at the Naval Air Warfare Center Weapons Division, China Lake, California, from October 2001 to September 2004. It was funded by the Precision Strike Navigator project sponsored by the Office of Naval Research.

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(U) In this paper we derive an expression for the output intensity of a general Interferometric Fiber Optic Gyroscope (IFOG). The light source is assumed to be non-monocromatic with an arbitrary frequency distribution function belonging to $L^2(R)$. This result is expanded and expressed in terms of the parameters involved in the definition of each of the four entries of the clockwise Jones matrix of a generic IFOG with an even frequency distribution. We obtain an expression for the Sagnac phase shift error and compute it for a Gaussian, rectangular, and triangular frequency distribution function. The clockwise loop Jones matrix of a fairly general IFOG model is computed. Matlab programs for computing the output intensity and Sagnac phase shift error are also included.

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CONTENTS

1. INTRODUCTION

The importance of polarization control is increasingly being recognized in the development of optical fiber gyroscopes. In Reference 1, Kintner presented a concise analysis of a very basic interferometric fiber optic gyroscope (IFOG) arrangement that demonstrates how the lack of perfect polarizers contributes to an error in the estimate of the Sagnac phase shift due to polarization. His design is an idealized design that does not include a model of any of the possible misalignments in the various optical fiber junctions involved. This simplification leads to a very simple model that allows him to derive a close form expression for the output intensity where the ideal gyro output and the error term due to imperfect polarizers are readily identifiable. Thus, he is able to estimate the most unfavorable error in the Sagnac phase shift due to polarization.

In this paper we consider a more general and realistic design that includes a model of the misalignment errors at the various optical fiber junctions involved. It also includes a model of the integrated optics chips and depolarizers as well as a general model for the coil. We obtain a closed form expression for the output intensity that includes the effects of the light source coherence. From this expression the ideal gyro output can be deduced and an expression for the Sagnac phase shift error due to imperfect polarizers is derived.

In Section 2 we obtain an expression for the output intensity of a general IFOG. The light source is assumed to be non-monocromatic with an arbitrary frequency distribution function belonging to $L^2(R)$. This result is expanded in Section 3 and expresses in terms of the four entries of the 2×2 complex matrix representing the Jones matrix of the clockwise loop and further, in terms of the parameters involved in the definition of each of the four entries of the clockwise Jones matrix for a generic IFOG with an even frequency distribution function. At this point the Sagnac phase shift error is derived in Section 3.1. In Sections 3.2, 3.3, and 3.4 we compute the output intensity for three

particular frequency distributions: a Gaussian distribution, a rectangular distribution, and a triangular distribution. The theory developed in these sections is illustrated with a simple example in Section 4 for the case of a rectangular frequency distribution.

The effects of the light source coherence on the Sagnac error are demonstrated in Section 4.2 by varying the length of a particular section of PM-fiber. We show plots of the Sagnac error angle as a function of fiber length for five different models of the coil. Section 4.3 includes a short discussion on how one might model modulation effects.

The Jones matrices of the components of a more general IFOG model are described in Section 5. The clockwise and counter clockwise loop matrices are defined in terms of products of the Jones matrices of the components involved. The complete computation of the clockwise loop matrix is carried out in Appendix A. The end result are the four arrays of 256 coefficients and four arrays of 256 sets of indices needed to define the four entries of the clockwise IFOG Jones matrix as defined by Equation 3.3.

We wrote Matlab programs to compute the output intensities given by Equations 3.14, 3.15, and 3.19 and to compute the Sagnac error given by Equation 3.13 for Gaussian, rectangular, and triangular frequency distribution functions. These programs are described in Appendix B where we include their listings. The plots in Figures 7, 8, and 9 were produced using these programs.

2. OUTPUT INTENSITY

Each component of the gyro has a matrix representation, it's Jones matrix (Reference 2). The clockwise loop of the gyro is represented by a matrix T which is obtained by multiplying the Jones matrices of all the components of the gyro in the proper order. The counter clockwise loop of the gyro is represented by the matrix $T_c = e^{i\phi} \cdot T^T$, where T^T is the transpose of the clockwise loop matrix T and ϕ is the

Sagnac phase shift due to rotation. Since some of the optical components of the gyro are frequency dependent, the matrix T is also frequency dependent. The sum of the clockwise and counter-clockwise loop matrices is denoted by Ω. Thus, $\Omega(\omega) = T(\omega) + e^{i\phi}T^T(\omega)$, where ω denotes the frequency dependence of the optical components of the gyro.

We will consider an input of the form

$$e(t) = \mathrm{Re}\left[\bar{a}\int_{-\infty}^{\infty} g(\omega - \omega_o)e^{-i\omega t}d\omega\right] = \mathrm{Re}\left[\bar{a}\int_{-\infty}^{\infty} g(\omega)e^{-i(\omega+\omega_o)t}d\omega\right]$$

where Re denotes the real part, $\bar{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ is a two dimensional complex vector of norm 1; that is, $|a_1|^2 + |a_2|^2 = 1$. In the first part of our analysis, the frequency distribution function g can be any function in $L^2(R)$. Later we specialize g to be a step function, a tent function, or a Gaussian function of the form $g(\omega) = \frac{1}{\sigma\sqrt{\pi}}e^{-\omega^2/\sigma^2}$ that has been normalized so that $\int_{-\infty}^{\infty} g(\omega)d\omega = 1$.

Let $\Theta(t) = \int_{-\infty}^{\infty} \Omega(\omega)\bar{a}g(\omega)e^{-i(\omega+\omega_o)t}d\omega$. Then, the output of the gyro is given by $\mathrm{Re}\,\Theta(t)$.

Theorem.

(a) If the output intensity is given by $I_C = \int_{-\infty}^{\infty} \Theta^*(t)\Theta(t)\,dt$, then

$$I_C = 2\pi \int_{-\infty}^{\infty} \left\| \Omega(\omega)\bar{a} \right\|^2 g^2(\omega) d\omega .$$

(b) If the output intensity is given by $I_R = \int_{-\infty}^{\infty} [\mathrm{Re}\,\Theta(t)]^T \,\mathrm{Re}\,\Theta(t)\, dt$ and g is a real valued function, then

$$I_R = \tfrac{1}{2} I_C + \pi \int_{-\infty}^{\infty} \mathrm{Re}\left[\bar{a}^T \Omega^T (u-\omega_o)\Omega(-u-\omega_o)\bar{a}\right] g(u-\omega_o) g(-u-\omega_o) du .$$

Proof.

Part (a) is based on the following identity satisfied by any function $f \in L^2(R)$.

$$f(t) = \tfrac{1}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(s) e^{\pm i(s-t)\omega} ds\, d\omega . \qquad (2.1)$$

which is simply the Fourier inversion formula.

Substituting $\Theta(t)$ in the Integral defining I_C gives

$$I_C = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} \bar{a}^* \Omega^* (\omega) g(\omega) e^{i(\omega+\omega_o)t} d\omega \right] \left[\int_{-\infty}^{\infty} \Omega(s)\bar{a} g(s) e^{-i(s+\omega_o)t} ds \right] dt .$$

Combining the two inner integrals as a double integral gives

$$I_C = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \bar{a}^* \Omega^* (\omega) g(\omega) \Omega(s) \bar{a}\, g(s) e^{-i(s-\omega)t} d\omega ds \right] dt .$$

Writing I as a triple integral and switching the order of integration gives

$$I_C = \int_{-\infty}^{\infty} \bar{a}^* \Omega^*(\omega) g(\omega) \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Omega(s) \bar{a} g(s) e^{-i(s-\omega)t} ds\, dt \right] d\omega .$$

Applying the Identity 2.1 to the inner double integral (note that t and ω are switched) gives

$$I_C = 2\pi \int_{-\infty}^{\infty} \bar{a}^* \Omega^*(\omega) g(\omega) \Omega(\omega) \bar{a} g(\omega) d\omega = 2\pi \int_{-\infty}^{\infty} \left\| \Omega(\omega) \bar{a} \right\|^2 g^2(\omega) d\omega .$$

Part (b) is based on the following identity satisfied by any function $f : R \to R$ belonging to $L^2(R)$.

$$f(\omega) = \tfrac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(s) \cos(\omega - s)t\; ds\; dt . \qquad (2.2)$$

Note that Equation 2.2 is simply the Fourier inversion formula for a real valued function f.

First we compute $\operatorname{Re}\Theta(t)$.

$$\operatorname{Re}\Theta(t) = \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(s)\bar{a}] g(s) \cos(s + \omega_o)t\; ds \; - \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(s)\bar{a}] g(s) \sin(s + \omega_o)t\; ds$$

$$[\operatorname{Re}\Theta(t)]^T = \int_{-\infty}^{\infty} \operatorname{Re}[\Omega(\omega)\bar{a}]^T g(\omega) \cos(\omega + \omega_o)t\; d\omega \; - \int_{-\infty}^{\infty} \operatorname{Im}[\Omega(\omega)\bar{a}]^T g(\omega) \sin(\omega + \omega_o)t\; d\omega$$

$$
\begin{aligned}
I_R = & \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(\omega)\vec{a}]^T g(\omega)\cos(\omega+\omega_o)t\, d\omega\right]\left[\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(s)\vec{a}]g(s)\cos(s+\omega_o)t\, ds)\right]dt\ + \\
& -\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(\omega)\vec{a}]^T g(\omega)\cos(\omega+\omega_o)t\, d\omega\right]\left[\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(s)\vec{a}]g(s)\sin(s+\omega_o)t\, ds)\right]dt\ + \\
& -\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(\omega)\vec{a}]^T g(\omega)\sin(\omega+\omega_o)t\, d\omega\right]\left[\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(s)\vec{a}]g(s)\cos(s+\omega_o)t\, ds)\right]dt\ + \\
& \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(\omega)\vec{a}]^T g(\omega)\sin(\omega+\omega_o)t\, d\omega\right]\left[\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(s)\vec{a}]g(s)\sin(s+\omega_o)t\, ds)\right]dt\,.
\end{aligned}
$$

If we write the above integrals as a triple integral and integrate with respect to t first, the second and third integrals vanish because $\cos(\omega+\omega_o)t\sin(s+\omega_o)t$ and $\sin(\omega+\omega_o)t\cos(s+\omega_o)t$ are odd functions of t. Thus, we are left with the first and fourth integrals which we write as

$$
\begin{aligned}
I_R = & \int_{-\infty}^{\infty}\mathrm{Re}[\Omega(\omega)\vec{a}]^T g(\omega)\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(s)\vec{a}]g(s)\cos(\omega+\omega_o)t\cos(s+\omega_o)t\, ds\, dt\right]d\omega\ + \\
& \int_{-\infty}^{\infty}\mathrm{Im}[\Omega(\omega)\vec{a}]^T g(\omega)\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(s)\vec{a}]g(s)\sin(\omega+\omega_o)t\sin(s+\omega_o)t\, ds\, dt\right]d\omega\,.
\end{aligned}
$$

The identities $\cos(\omega+\omega_o)t\cos(s+\omega_o)t=\frac{1}{2}[\cos(\omega-s)t+\cos(\omega+s+2\omega_o)t]$ and $\sin(\omega+\omega_o)t\sin(s+\omega_o)t=\frac{1}{2}[\cos(\omega-s)t-\cos(\omega+s+2\omega_o)t]$ yield

$$
\begin{aligned}
I_R = & \tfrac{1}{2}\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(\omega)\vec{a}]^T g(\omega)\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(s)\vec{a}]g(s)\cos(\omega-s)t\, ds\, dt\right]d\omega\ + \\
& \tfrac{1}{2}\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(\omega)\vec{a}]^T g(\omega)\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{Re}[\Omega(s)\vec{a}]g(s)\cos(\omega+2\omega_o+s)t\, ds\, dt\right]d\omega\ + \\
& \tfrac{1}{2}\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(\omega)\vec{a}]^T g(\omega)\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(s)\vec{a}]g(s)\cos(\omega-s)t\, ds\, dt\right]d\omega\ + \\
& -\tfrac{1}{2}\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(\omega)\vec{a}]^T g(\omega)\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{Im}[\Omega(s)\vec{a}]g(s)\cos(\omega+2\omega_o+s)t\, ds\, dt\right]d\omega\,.
\end{aligned}
$$

The Identity 2.2 gives

$$
\begin{aligned}
I_R = {} & \pi \int_{-\infty}^{\infty} \mathrm{Re}[\Omega(\omega)\vec{a}]^T \,\mathrm{Re}[\Omega(\omega)\vec{a}] g^2(\omega)\, d\omega + \pi \int_{-\infty}^{\infty} \mathrm{Im}[\Omega(\omega)\vec{a}]^T \,\mathrm{Im}[\Omega(\omega)\vec{a}] g^2(\omega)\, d\omega + \\
& \pi \int_{-\infty}^{\infty} \mathrm{Re}[\Omega(\omega)\vec{a}]^T \,\mathrm{Re}[\Omega(-\omega - 2\omega_o)\vec{a}] g(\omega) g(-\omega - 2\omega_o)\, d\omega + \\
& -\pi \int_{-\infty}^{\infty} \mathrm{Im}[\Omega(\omega)\vec{a}]^T \,\mathrm{Im}[\Omega(-\omega - 2\omega_o)\vec{a}] g(\omega) g(-\omega - 2\omega_o)\, d\omega .
\end{aligned}
$$

The identities $\|a\|^2 = a^* a = \mathrm{Re}\, a^T \cdot \mathrm{Re}\, a + \mathrm{Im}\, a^T \cdot \mathrm{Im}\, a$ and $\mathrm{Re}[a^T c] = \mathrm{Re}\, a^T \cdot \mathrm{Re}\, c - \mathrm{Im}\, a^T \cdot \mathrm{Im}\, c$, which hold for any two complex vectors a and c, yield

$$
I_R = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\vec{a}\|^2 g^2(\omega)\, d\omega + \pi \int_{-\infty}^{\infty} \mathrm{Re}\big[\vec{a}^T \Omega^T(\omega)\Omega(-\omega - 2\omega_o)\vec{a}\big] g(\omega) g(-\omega - 2\omega_o) d\omega ,
$$

or

$$
I_R = \tfrac{1}{2} I_C + \pi \int_{-\infty}^{\infty} \mathrm{Re}\big[\vec{a}^T \Omega^T(u - \omega_o)\Omega(-u - \omega_o)\vec{a}\big] g(u - \omega_o) g(-u - \omega_o) du ,
$$

where in the last integral we have made the change of variables $u = \omega + \omega_o$.

Corollary. If g is *zero* outside an interval $[-\omega_1, \omega_1]$ and $\omega_1 < \omega_o$, then the product $g(u - \omega_o) g(-u - \omega_o) \equiv 0$ for all $u \in R$, consequently,

$$
I_R = \tfrac{1}{2} I_C = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\vec{a}\|^2 g^2(\omega) d\omega . \tag{2.3}
$$

Proof. If g is *zero* outside the interval $[-\omega_1, \omega_1]$, then $g(u - \omega_o)$ can be *non zero* only when $\omega_o - \omega_1 \le u \le \omega_o + \omega_1$, and $g(-u - \omega_o)$ can be *non zero* only when

$-\omega_o - \omega_1 \le u \le \omega_1 - \omega_o$. Since these two sets do not intersect, because $\omega_1 - \omega_o < 0 < \omega_o - \omega_1$, the product $g(u-\omega_0)g(-u-\omega_0)$ is zero for all $u \in R$.

3. OUTPUT INTENSITY IN TERMS OF THE MATRIX *T* AND IN TERMS OF THE PARAMETERS DEFINING THE IFOG

In this section we obtain a closed form expression for the output intensity of the gyro in terms of the four entries of the complex 2×2 matrix *T* and in terms of the parameters involved in the four entries of the matrix *T* for three types of function g: a step function, a tent function, and a Gaussian function.

Part (a) of Theorem 1 holds for any function $g \in L^2(R)$ while part (b) of Theorem 1 holds for any *real valued* function $g \in L^2(R)$. When g is a step or a tent function, Corollary 1 will apply and the output intensity will be given by Equation 2.3. When g is the Gaussian function defined in Section 2, $g(u-\omega_o)g(-u-\omega_o) = g(\sqrt{2}\omega_o)g(\sqrt{2}u)$ for all $u \in R$, so we can write I_R as

$$I_R = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\vec{a}\|^2 g^2(\omega)d\omega + \pi g(\sqrt{2}\omega_o) \int_{-\infty}^{\infty} \mathrm{Re}\left[\vec{a}^T \Omega^T (u-\omega_o)\Omega(-u-\omega_o)\vec{a}\right] g(\sqrt{2}u)du \, .$$

Since ω_o/σ is a very large number, $g(\sqrt{2}\omega_o)$ is a very small number, thus the second integral above can be neglected. Therefore, by Equation 2.3, in all three cases it will suffice to compute I_C.

We start by defining expressions for the four entries of *T* and compute $\|\Omega\vec{a}\|^2$.

If we write T as $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$, then

$$\Omega\vec{a} = [T + e^{i\phi}T^T]\vec{a} = \begin{bmatrix} T_{11} + e^{i\phi}T_{11} & T_{12} + e^{i\phi}T_{21} \\ T_{21} + e^{i\phi}T_{12} & T_{22} + e^{i\phi}T_{22} \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} (T_{11} + e^{i\phi}T_{11})a_1 + (T_{12} + e^{i\phi}T_{21})a_2 \\ (T_{21} + e^{i\phi}T_{12})a_1 + (T_{22} + e^{i\phi}T_{22})a_2 \end{bmatrix} \tag{3.1}$$

Hence,

$$\begin{aligned} \|\Omega(\omega)\vec{a}\|^2 \;=\; & \vec{a}^*\Omega^*(\omega)\Omega(\omega)\vec{a} = \\ & 2|a_1|^2(1+\cos\phi)|T_{11}|^2 + 2|a_2|^2(1+\cos\phi)|T_{22}|^2 + |T_{12}|^2 + |T_{21}|^2 + \\ & 2|a_1|^2 \operatorname{Re}[\overline{T}_{21}T_{12}e^{i\phi}] + 2|a_2|^2 \operatorname{Re}[\overline{T}_{12}T_{21}e^{i\phi}] + \\ & 2\operatorname{Re}\{a_1\overline{a}_2[\overline{T}_{12}T_{11} + \overline{T}_{12}e^{i\phi}T_{11}]\} + 2\operatorname{Re}\{a_1\overline{a}_2[e^{-i\phi}\overline{T}_{21}T_{11} + \overline{T}_{21}T_{11}]\} + \\ & 2\operatorname{Re}\{a_1\overline{a}_2[\overline{T}_{22}T_{21} + e^{-i\phi}\overline{T}_{22}T_{21}]\} + 2\operatorname{Re}\{a_1\overline{a}_2[\overline{T}_{22}e^{i\phi}T_{12} + \overline{T}_{22}T_{12}]\}. \end{aligned} \tag{3.2}$$

Each T_{ij} is a sum of 2^N terms, where N is the number of rotations (miss-alignments) in the fiber splicings. They have the form (see Section 4)

$$T_{ij}(\omega) = \sum_{s=1}^{2^N} C_{ij}(s)\exp i(\omega \sum_{r\in\Lambda_{ij}(s)} n_r\ell_r / c)\,. \tag{3.3}$$

Thus, the factor $\|\Omega(\omega)\vec{a}\|^2$ in Theorem 1 involves *thousands* of terms if $N \geq 4$. Each C_{ij} is a 2^N-array of coefficients and each $\Lambda_{ij}(s)$ is set of indices, (s = 1, 2, ..., 2^N; i, j = 1, 2). Let $\beta_{ij}(s)$ be given by

$$\beta_{ij}(s) \equiv \sum_{r \in \Lambda_{ij}(s)} n_r \ell_r / c, \text{ for } s = 1,2,\ldots,2^N \text{ and } i,j = 1,2 \tag{3.4}$$

and let $T_{uv}(\omega) = \sum_{p=1}^{2^N} C_{uv}(p) e^{i\beta_{uv}(p)\omega}$ and $a_1\overline{a}_2 = |a_1\overline{a}_2| e^{i\alpha}$. Then,

$$a_1\overline{a}_2 e^{\pm i\phi} \overline{T}_{ij}(\omega) T_{uv}(\omega) = |a_1\overline{a}_2| \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{uv}(p) e^{i\{\alpha \pm \phi + [\beta_{uv}(p) - \beta_{ij}(s)]\omega\}}$$

$$\mathrm{Re}\left[a_1\overline{a}_2 e^{\pm i\phi} \overline{T}_{ij}(\omega) T_{uv}(\omega)\right] = |a_1\overline{a}_2| \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{uv}(p) \cos\{\alpha \pm \phi + [\beta_{uv}(p) - \beta_{ij}(s)]\omega\}$$

and

$$\left|T_{ij}(\omega)\right|^2 = \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{ij}(p) \cos[\beta_{ij}(p) - \beta_{ij}(s)]\omega .$$

Thus,

$$\int_{-\infty}^{\infty} \left|T_{ij}(\omega)\right|^2 g^2(\omega)\, d\omega = \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} C_{ij}(s) C_{ij}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos\{[\beta_{ij}(p) - \beta_{ij}(s)]\omega\}\, d\omega \tag{3.5}$$

and

$$\int_{-\infty}^{\infty} \mathrm{Re}\left[a_1\overline{a}_2 e^{\pm i\phi} \overline{T}_{ij}(\omega) T_{uv}(\omega)\right] g^2(\omega)\, d\omega =$$

$$|a_1\bar{a}_2|\sum_{s=1}^{2^N}\sum_{p=1}^{2^N}C_{ij}(s)C_{uv}(p)\int_{-\infty}^{\infty}g^2(\omega)\cos\{\alpha\pm\phi+[\beta_{uv}(p)-\beta_{ij}(s)]\omega\}\,d\omega\,, \quad (3.6)$$

where g in Equations 3.5 and 3.6 can be any function in L^2.

Since

$$\cos\{\alpha\pm\phi+[\beta_{uv}(p)-\beta_{ij}(s)]\omega\}=$$
$$\cos(\alpha\pm\phi)\cos[\beta_{uv}(p)-\beta_{ij}(s)]\omega-\sin(\alpha\pm\phi)\sin[\beta_{uv}(p)-\beta_{ij}(s)]\omega\,,$$

when the function g is an *even* function Equation 3.6 simplifies to

$$\int_{-\infty}^{\infty}\mathrm{Re}\left[a_1\bar{a}_2e^{\pm i\phi}\overline{T}_{ij}(\omega)T_{uv}(\omega)\right]g^2(\omega)\,d\omega=$$
$$|a_1\bar{a}_2|\cos(\alpha\pm\phi)\sum_{s=1}^{2^N}\sum_{p=1}^{2^N}C_{ij}(s)C_{uv}(p)\int_{-\infty}^{\infty}g^2(\omega)\cos[\beta_{uv}(p)-\beta_{ij}(s)]\omega\,d\omega\,, \quad (3.7)$$

which implies that when g is *even*

$$\int_{-\infty}^{\infty}\mathrm{Re}\left[a_1\bar{a}_2e^{\pm i\phi}\overline{T}_{ij}(\omega)T_{uv}(\omega)\right]g^2(\omega)\,d\omega=|a_1\bar{a}_2|\cos(\alpha\pm\phi)\int_{-\infty}^{\infty}\mathrm{Re}\left[\overline{T}_{ij}(\omega)T_{uv}(\omega)\right]g^2(\omega)\,d\omega\,. \quad (3.8)$$

We can now put together Part (a) of Theorem 1 with Equations 3.2 and 3.8 to obtain an expression for the output intensity in terms of $\int_{-\infty}^{\infty}\mathrm{Re}\left[\overline{T}_{ij}(\omega)T_{uv}(\omega)\right]g^2(\omega)\,d\omega$, for *i, j, u,* v = 1 or 2 when the function g is an even function.

$$
\begin{aligned}
I_C = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega)\vec{a}\|^2 g^2(\omega)\, d\omega = \\
2\pi\{\ & 2|a_1|^2(1+\cos\phi) \int_{-\infty}^{\infty} |T_{11}|^2 g^2 d\omega + 2|a_2|^2(1+\cos\phi) \int_{-\infty}^{\infty} |T_{22}|^2 g^2 d\omega + \\
& \int_{-\infty}^{\infty} |T_{12}|^2 g^2 d\omega + \int_{-\infty}^{\infty} |T_{21}|^2 g^2 d\omega + 2\cos\phi \int_{-\infty}^{\infty} \mathrm{Re}[\overline{T}_{21} T_{12}] g^2 d\omega + \\
& 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha+\phi)] \int_{-\infty}^{\infty} \mathrm{Re}(\overline{T}_{12} T_{11}) g^2 d\omega + \\
& 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha-\phi)] \int_{-\infty}^{\infty} \mathrm{Re}(\overline{T}_{21} T_{11}) g^2 d\omega + \\
& 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha-\phi)] \int_{-\infty}^{\infty} \mathrm{Re}(\overline{T}_{22} T_{21}) g^2 d\omega + \\
& 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha+\phi)] \int_{-\infty}^{\infty} \mathrm{Re}(\overline{T}_{22} T_{12}) g^2 d\omega \ \}.
\end{aligned}
$$

More explicitly, in terms of the integrals

$$
\int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{ij}(s) - \beta_{uv}(p)]\omega\, d\omega, \tag{3.9}
$$

when the function g is an even function using Equations 3.5 and 3.7 we get,

$$
\begin{aligned}
I_C = 2\pi \int_{-\infty}^{\infty} \|\Omega(\omega)\vec{a}\|^2 g^2(\omega)\, d\omega = \\
2\pi \sum_{s=1}^{2^N} \sum_{p=1}^{2^N} \{\ 2|a_1|^2(1+\cos\phi) C_{11}(s) C_{11}(p) \int_{-\infty}^{\infty} g^2(\omega) \cos[\beta_{11}(p) - \beta_{11}(s)]\omega\, d\omega +
\end{aligned}
$$

$$2|a_2|^2(1+\cos\phi)C_{22}(s)C_{22}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{22}(p)-\beta_{22}(s)]\omega\, d\omega \;+$$

$$C_{12}(s)C_{12}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{12}(p)-\beta_{12}(s)]\omega\, d\omega \;+$$

$$C_{21}(s)C_{21}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{21}(p)-\beta_{21}(s)]\omega\, d\omega \;+$$

$$2\cos\phi\, C_{12}(s)C_{21}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{12}(p)-\beta_{21}(s)]\omega\, d\omega \;+$$

$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha+\phi)]C_{12}(s)C_{11}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{12}(p)-\beta_{11}(s)]\omega\, d\omega \;+$$

$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha-\phi)]C_{21}(s)C_{11}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{21}(p)-\beta_{11}(s)]\omega\, d\omega \;+$$

$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha-\phi)]C_{22}(s)C_{21}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{22}(p)-\beta_{21}(s)]\omega\, d\omega \;+$$

$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha+\phi)]C_{22}(s)C_{12}(p)\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{22}(p)-\beta_{12}(s)]\omega\, d\omega \;\}. \tag{3.10}$$

3.1. THE SAGNAC PHASE SHIFT ERROR

In this Section we derive the Sagnac phase shift error due to polarization. Let $I_o(\phi)$ denote the output intensity of the gyro with perfect polarizers (attenuating coefficient ε = 0) as a function of the Sagnac phase shift ϕ and let $I_\varepsilon(\phi)$ denote the output intensity of the gyro with imperfect polarizers ($\varepsilon > 0$). When the polarizers are perfect, $T_{12} = 0$, $T_{21} = 0$, and $T_{22} = 0$ (see the examples in Sections 4 and 5). Thus,

$$I_o(\phi) = 4\pi\; |a_1|^2(1+\cos\phi)\int_{-\infty}^{\infty} |T_{11}|^2 g^2 d\omega .$$

Since $I_o(-\pi/2) = 4\pi \; |a_1|^2 \int_{-\infty}^{\infty} |T_{11}|^2 g^2 d\omega$, we can write $I_o(\phi)$ as

$$I_o(\phi) = I_o(-\pi/2)\,(1+\cos\phi)\,. \tag{3.11}$$

The difference between $I_\varepsilon(\phi)$ and $I_o(\phi)$ is the error induced by having an imperfect polarizer. This error induces an error in the estimate of the Sagnac phase shift ϕ that we will call ϕ_ε. It can be calculated by comparing $I_\varepsilon(-\pi/2)$ with $I_o(-\pi/2+\phi_\varepsilon)$. The idea is that there is a rotation angle ϕ_ε such that

$$I_\varepsilon(-\pi/2) = I_o(-\pi/2+\phi_\varepsilon)\,. \tag{3.12}$$

By Equation 3.11, $I_o(-\pi/2+\phi_\varepsilon) \;=\; I_o(-\pi/2)\,[1+\cos(-\pi/2+\phi_\varepsilon)]$. Substituting in Equation 3.12 and solving for ϕ_ε we get,

$$\begin{aligned} I_\varepsilon(-\pi/2) &= I_o(-\pi/2)\,[1+\cos(-\pi/2+\phi_\varepsilon)] \\ &= I_o(-\pi/2)\,[1+\cos(\pi/2)\cos\phi_\varepsilon+\sin(\pi/2)\sin\phi_\varepsilon] \\ &= I_o(-\pi/2)\,[1+\sin\phi_\varepsilon] \end{aligned}$$

$$\Rightarrow \qquad \phi_\varepsilon = \sin^{-1}\left(\frac{I_\varepsilon(-\pi/2)}{I_o(-\pi/2)}-1\right). \tag{3.13}$$

3.2. GAUSSIAN FUNCTION *g*

According to Equation 3.10, all we need to obtain the intensity I_C for an even function g are the integrals $\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega\, d\omega$.

When the function g is the Gaussian function defined in Section 2, which is an even function, we use the following identity (Reference 3).

$$\int_{-\infty}^{\infty} e^{-x^2/q^2}\cos[p(x+\lambda)]dx = q\sqrt{\pi}e^{-p^2q^2/4}\cos p\lambda\,,$$

from which we get,

$$\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega\, d\omega =$$

$$\tfrac{1}{\sigma^2\pi}\int_{-\infty}^{\infty} e^{-2\omega^2/\sigma^2}\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega\, d\omega = \tfrac{1}{\sigma\sqrt{2\pi}}e^{-[\beta_{ij}(s)-\beta_{uv}(p)]^2\sigma^2/8}\,,$$

thus, by Equation 3.10

$$I_C = 2\pi\int_{-\infty}^{\infty}\left\|\Omega(\omega)\vec{a}\right\|^2 g^2(\omega)\, d\omega =$$

$$\frac{\sqrt{2\pi}}{\sigma}\sum_{s=1}^{2^N}\sum_{p=1}^{2^N}\{\ 2|a_1|^2(1+\cos\phi)C_{11}(s)C_{11}(p)e^{-[\beta_{11}(s)-\beta_{11}(p)]^2\sigma^2/8} +$$
$$2|a_2|^2(1+\cos\phi)C_{22}(s)C_{22}(p)e^{-[\beta_{22}(s)-\beta_{22}(p)]^2\sigma^2/8} +$$
$$C_{12}(s)C_{12}(p)e^{-[\beta_{12}(s)-\beta_{12}(p)]^2\sigma^2/8} +$$
$$C_{21}(s)C_{21}(p)e^{-[\beta_{21}(s)-\beta_{21}(p)]^2\sigma^2/8} +$$
$$2\cos\phi\, C_{12}(s)C_{21}(p)e^{-[\beta_{12}(s)-\beta_{21}(p)]^2\sigma^2/8} +$$
$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha+\phi)]C_{12}(s)C_{11}(p)e^{-[\beta_{12}(s)-\beta_{11}(p)]^2\sigma^2/8} +$$
$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha-\phi)]C_{21}(s)C_{11}(p)e^{-[\beta_{21}(s)-\beta_{11}(p)]^2\sigma^2/8} +$$
$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha-\phi)]C_{22}(s)C_{21}(p)e^{-[\beta_{22}(s)-\beta_{21}(p)]^2\sigma^2/8} +$$
$$2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha+\phi)]C_{22}(s)C_{12}(p)e^{-[\beta_{22}(s)-\beta_{12}(p)]^2\sigma^2/8}\ \}. \tag{3.14}$$

3.3. STEP FUNCTION g

In this Section we let the frequency distribution function be a step function g_S. Let g_S be defined as

$$g_S(\omega)=\begin{cases}\frac{1}{2\omega_1} & \text{for } |\omega|\le\omega_1\\ 0 & \text{otherwise}\end{cases},\quad \text{with } 0<\omega_1<\omega_o .$$

Since g_S is an even function, we only need the integrals $\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega\, d\omega$. We use the following result.

$$\int_{-\infty}^{\infty} g_S{}^2(x)\cos\beta x\,dx = \frac{1}{4\omega_1^2}\int_{-\omega_1}^{\omega_1}\cos\beta x\,dx = \frac{1}{2\omega_1}\frac{\sin\beta\omega_1}{\beta\omega_1},$$

from which we get,

$$\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega\, d\omega =$$

$$\frac{1}{4\omega_1^2}\int_{-\infty}^{\infty}\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega\, d\omega = \frac{1}{2\omega_1}\frac{\sin[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1}{[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1},$$

thus, by Equation 3.10

$$\begin{aligned}
I_C = \frac{\pi}{2\omega_1}\sum_{s=1}^{2^N}\sum_{p=1}^{2^N} \Big\{ & 2|a_1|^2\,(1+\cos\phi)C_{11}(s)C_{11}(p)\frac{\sin[\beta_{11}(s)-\beta_{11}(p)]\omega_1}{[\beta_{11}(s)-\beta_{11}(p)]\omega_1} + \\
& 2|a_2|^2\,(1+\cos\phi)C_{22}(s)C_{22}(p)\frac{\sin[\beta_{22}(s)-\beta_{22}(p)]\omega_1}{[\beta_{22}(s)-\beta_{22}(p)]\omega_1} + \\
& C_{12}(s)C_{12}(p)\frac{\sin[\beta_{12}(s)-\beta_{12}(p)]\omega_1}{[\beta_{12}(s)-\beta_{12}(p)]\omega_1} + \\
& C_{21}(s)C_{21}(p)\frac{\sin[\beta_{21}(s)-\beta_{21}(p)]\omega_1}{[\beta_{21}(s)-\beta_{21}(p)]\omega_1} + \\
& 2\cos\phi\, C_{12}(s)C_{21}(p)\frac{\sin[\beta_{12}(s)-\beta_{21}(p)]\omega_1}{[\beta_{12}(s)-\beta_{21}(p)]\omega_1} + \\
& 2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha+\phi)]C_{11}(s)C_{12}(p)\frac{\sin[\beta_{11}(s)-\beta_{12}(p)]\omega_1}{[\beta_{11}(s)-\beta_{12}(p)]\omega_1} + \\
& 2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha-\phi)]C_{11}(s)C_{21}(p)\frac{\sin[\beta_{11}(s)-\beta_{21}(p)]\omega_1}{[\beta_{11}(s)-\beta_{21}(p)]\omega_1} + \\
& 2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha-\phi)]C_{21}(s)C_{22}(p)\frac{\sin[\beta_{21}(s)-\beta_{22}(p)]\omega_1}{[\beta_{21}(s)-\beta_{22}(p)]\omega_1} + \\
& 2|a_1\bar{a}_2|[\cos\alpha+\cos(\alpha+\phi)]C_{22}(s)C_{12}(p)\frac{\sin[\beta_{22}(s)-\beta_{12}(p)]\omega_1}{[\beta_{22}(s)-\beta_{12}(p)]\omega_1} \Big\}.
\end{aligned} \tag{3.15}$$

3.4. TENT FUNCTION g

The analysis above can be carried out when the function g is a "tent" function g_T defined as

$$g_T(x) \equiv \frac{1}{\omega_1}\begin{cases} 0 & for \quad |x| > \omega_1 \\ 1+\dfrac{1}{\omega_1}x & for \quad -\omega_1 \le x \le 0 \\ 1-\dfrac{1}{\omega_1}x & for \quad 0 \le x \le \omega_1 \end{cases}$$

Since g_T is even, we only need $\int_{-\infty}^{\infty} g^2(\omega)\cos[\beta_{ij}(s)-\beta_{uv}(p)]\omega \, d\omega$. We will use the next three formulas:

$$\int 1 \cdot \cos \beta x \, dx = \frac{1}{\beta}\sin \beta x$$

$$\int x \cdot \cos \beta x \, dx = \frac{1}{\beta^2}\cos \beta x + \frac{1}{\beta} x \sin \beta x$$

$$\int x^2 \cdot \cos \beta x \, dx = -\frac{2}{\beta^3}\sin \beta x + \frac{2}{\beta^2} x \cos \beta x + \frac{1}{\beta} x^2 \sin \beta x .$$

From the definition of g_T we have

$$g_T^{\,2}(x) \equiv \frac{1}{\omega_1^2}\begin{cases} 0 & for \quad |x| > \omega_1 \\ 1+\dfrac{2}{\omega_1}x+\dfrac{1}{\omega_1^2}x^2 & for \quad \omega_1 \le x \le 0 \\ 1-\dfrac{2}{\omega_1}x+\dfrac{1}{\omega_1^2}x^2 & for \quad 0 \le x \le \omega_1 \end{cases}$$

When $\beta \neq 0$

$$\int_{-\infty}^{\infty} g_T^{\,2}(x)\cos\beta x\,dx = \frac{2}{\omega_1^2}\int_0^{\omega_1}(1-\frac{2}{\omega_1}x+\frac{1}{\omega_1^2}x^2)\cos\beta x\,dx =$$

$$= \frac{2}{\omega_1^2\beta}\left[1-\frac{2x}{\omega_1}+\frac{x^2}{\omega_1^2}-\frac{2}{\omega_1^2\beta^2}\right]\sin\beta x + \frac{2}{\omega_1^2\beta^2}\left[-\frac{2}{\omega_1}+\frac{2x}{\omega_1^2}\right]\cos\beta x\,\Bigg|_{x=0}^{\omega_1}$$

$$= \frac{4}{\beta^2\omega_1^3}\left[1-\frac{\sin\beta\omega_1}{\beta\omega_1}\right]. \tag{3.16}$$

If $\beta = 0$, then

$$\int_{-\infty}^{\infty} g_T^{\,2}(x)\,dx = \frac{2}{\omega_1^2}\int_0^{\omega_1}(1-\frac{2}{\omega_1}x+\frac{1}{\omega_1^2}x^2)\,dx = \frac{1}{\omega_1^2}(x-\frac{x^2}{\omega_1}+\frac{x^3}{3\omega_1^2})\Bigg|_0^{\omega_1} = \frac{2}{3\omega_1}. \tag{3.17}$$

Let $Z_{ij,uv} \equiv \{\,(s,p) : \beta_{ij}(p)-\beta_{uv}(s)=0\,\}$ and define

$$\Lambda_{ij,uv} \equiv \frac{2}{3\omega_1}\sum_{(s,p)\in Z_{uv,ij}} C_{ij}(s)C_{uv}(p) +$$
$$\frac{4}{\omega_1^3}\sum_{(s,p)\notin Z_{uv,ij}} \frac{C_{ij}(s)C_{uv}(p)}{[\beta_{ij}(p)-\beta_{uv}(s)]^2(s,p)}\left[1-\frac{\sin\{\omega_1[\beta_{ij}(p)-\beta_{uv}(s)]\}}{\omega_1[\beta_{ij}(p)-\beta_{uv}(s)]}\right]. \tag{3.18}$$

Note that $\Lambda_{ij,uv} = \int_{-\infty}^{\infty} \mathrm{Re}\left[\overline{T}_{ij}(\omega) T_{uv}(\omega)\right] g_T^2(\omega)\, d\omega$ by Equations 3.7, 3.16, and 3.17.

Then, by Equations 3.10, 3.16, 3.17, and the definition of $\Lambda_{ij,uv}$

$$\begin{aligned} I_C = 2\pi\{\ & 2|a_1|^2(1+\cos\phi)\Lambda_{11,11} + 2|a_2|^2(1+\cos\phi)\Lambda_{22,22} + \Lambda_{12,12} + \Lambda_{21,21} + \\ & 2\cos\phi\Lambda_{12,21} + 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha+\phi)]\Lambda_{12,11} + \\ & 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha-\phi)]\Lambda_{21,11} + 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha-\phi)]\Lambda_{22,21} + \\ & 2|a_1\overline{a}_2|[\cos\alpha + \cos(\alpha+\phi)]\Lambda_{22,12}\ \}. \end{aligned} \tag{3.19}$$

4. AN ELEMENTARY IFOG MODEL

As a first example we consider the simplest possible gyroscope. It consists of an integrated optics chip (IOC) connected to a length of polarization maintaining (PM) fiber at a 45° angle. The PM fiber is connected to a coil of single mode (SM) fiber which is then connected back to the IOC.

4.1. THE MODEL, OUTPUT INTENSITY, AND SAGNAC ERROR

The IOC is modeled by the Jones matrix

$$\mathrm{M} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \varepsilon\, e^{ik_6 l_3} \end{bmatrix}, \text{ with } k_5 = \tfrac{\omega}{c} n_5 \text{ and } k_6 = \tfrac{\omega}{c} n_6 .$$

The PM fiber of length l_5 and the 45° angle connection are modeled, respectively, by the Jones matrices

$$\mathrm{U} = \begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix}, \text{ with } k_9 = \tfrac{\omega}{c} n_9 ,\ k_{10} = \tfrac{\omega}{c} n_{10} , \text{ and } \mathrm{R}(45^{\circ}) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

The coil of SM fiber will be modeled by the matrix

$$\mathrm{C} = \begin{bmatrix} t_{11} e^{i\tau_{11}} & t_{12} e^{i\tau_{12}} \\ t_{21} e^{i\tau_{21}} & t_{22} e^{i\tau_{22}} \end{bmatrix}, \text{ with } \tau_{ij} = \tfrac{\omega}{c} n_{ij} l_o = k_{ij} l_o .$$

The clockwise loop matrix T is modeled by the product of the corresponding Jones matrices.

$$T(\omega) = \frac{1}{2\sqrt{2}}\begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \varepsilon\, e^{ik_6 l_3} \end{bmatrix}\begin{bmatrix} t_{11} e^{i\tau_{11}} & t_{12} e^{i\tau_{12}} \\ t_{21} e^{i\tau_{21}} & t_{22} e^{i\tau_{22}} \end{bmatrix}\begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \varepsilon\, e^{ik_6 l_3} \end{bmatrix}.$$

The product of the first three matrices is

$$\begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \varepsilon e^{ik_6 l_3} \end{bmatrix}\begin{bmatrix} t_{11}e^{i\tau_{11}} & t_{12}e^{i\tau_{12}} \\ t_{21}e^{i\tau_{21}} & t_{22}e^{i\tau_{22}} \end{bmatrix}\begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix} = \begin{bmatrix} t_{11}e^{i(\tau_{11}+k_5 l_3+k_9 l_5)} & t_{12}e^{i(\tau_{12}+k_5 l_3+k_{10} l_5)} \\ \varepsilon t_{21}e^{i(\tau_{21}+k_6 l_3+k_9 l_5)} & \varepsilon t_{22}e^{i(\tau_{22}+k_6 l_3+k_{10} l_5)} \end{bmatrix}.$$

Notice that the diagonal matrices modeling the IOC and the PM fiber do not increase the number of terms in the resulting matrix, they contribute to the number of terms in the complex exponential. However, a rotation in general will double the number of terms in the resulting matrix as can be seen next with the 45° rotation. This explains why, if the coil is modeled by a full Jones matrix, each T_{ij} is a sum of 2^N terms, where N is the number of rotations (missalignmets) in the fiber splicings along the clockwise loop.

$$\begin{bmatrix} t_{11}e^{i(\tau_{11}+k_5 l_3+k_9 l_5)} & t_{12}e^{i(\tau_{12}+k_5 l_3+k_{10} l_5)} \\ \varepsilon t_{21}e^{i(\tau_{21}+k_6 l_3+k_9 l_5)} & \varepsilon t_{22}e^{i(\tau_{22}+k_6 l_3+k_{10} l_5)} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} t_{11}e^{i(\tau_{11}+k_5 l_3+k_9 l_5)} - t_{12}e^{i(\tau_{12}+k_5 l_3+k_{10} l_5)} & [t_{11}e^{i(\tau_{11}+k_5 l_3+k_9 l_5)} + t_{12}e^{i(\tau_{12}+k_5 l_3+k_{10} l_5)}] \\ \varepsilon[t_{21}e^{i(\tau_{21}+k_6 l_3+k_9 l_5)} - t_{22}e^{i(\tau_{22}+k_6 l_3+k_{10} l_5)}] & \varepsilon[t_{21}e^{i(\tau_{21}+k_6 l_3+k_9 l_5)} + t_{22}e^{i(\tau_{22}+k_6 l_3+k_{10} l_5)}] \end{bmatrix}.$$

Finally, multiplying by the matrix modeling the IOC gives

$$T(\omega) = \frac{1}{2\sqrt{2}} \cdot$$

$$\begin{bmatrix} t_{11}e^{i(\tau_{11}+2k_5 l_3+k_9 l_5)} - t_{12}e^{i(\tau_{12}+2k_5 l_3+k_{10} l_5)} & \varepsilon[t_{11}e^{i(\tau_{11}+k_5 l_3+k_6 l_3+k_9 l_5)} + t_{12}e^{i(\tau_{12}+k_5 l_3+k_6 l_3+k_{10} l_5)}] \\ \varepsilon[t_{21}e^{i(\tau_{21}+k_5 l_3+k_6 l_3+k_9 l_5)} - t_{22}e^{i(\tau_{22}+k_5 l_3+k_6 l_3+k_{10} l_5)}] & \varepsilon^2[t_{21}e^{i(\tau_{21}+2k_6 l_3+k_9 l_5)} + t_{22}e^{i(\tau_{22}+2k_6 l_3+k_{10} l_5)}] \end{bmatrix}. \tag{4.1}$$

Thus, for the simple configuration of this example we have

$$T_{11}(\omega) = \tfrac{1}{2\sqrt{2}} t_{11} e^{i(\tau_{11}+2k_5l_3+k_9l_5)} - \tfrac{1}{2\sqrt{2}} t_{12} e^{i(\tau_{12}+2k_5l_3+k_{10}l_5)},$$

$$T_{12}(\omega) = \tfrac{1}{2\sqrt{2}} \varepsilon [t_{11} e^{i(\tau_{11}+k_5l_3+k_6l_3+k_9l_5)} + t_{12} e^{i(\tau_{12}+k_5l_3+k_6l_3+k_{10}l_5)}],$$

$$T_{21}(\omega) = \tfrac{1}{2\sqrt{2}} \varepsilon [t_{21} e^{i(\tau_{21}+k_5l_3+k_6l_3+k_9l_5)} - t_{22} e^{i(\tau_{22}+k_5l_3+k_6l_3+k_{10}l_5)}],$$

$$T_{22}(\omega) = \tfrac{1}{2\sqrt{2}} \varepsilon^2 [t_{21} e^{i(\tau_{21}+2k_6l_3+k_9l_5)} + t_{22} e^{i(\tau_{22}+2k_6l_3+k_{10}l_5)}],$$

and the $C_{ij}(s)$ and $\beta_{ij}(s)$ for $s = 1, 2$ representing the coefficients and the phases in the expressions for $T_{ij}(\omega)$ (see Equations 3.3 and 3.4) are

$$\begin{aligned}
C_{11}(1) &= \tfrac{1}{2\sqrt{2}} t_{11}, & C_{11}(2) &= -\tfrac{1}{2\sqrt{2}} t_{12},\\
C_{12}(1) &= \tfrac{1}{2\sqrt{2}} \varepsilon t_{11}, & C_{12}(2) &= \tfrac{1}{2\sqrt{2}} \varepsilon t_{12},\\
C_{21}(1) &= \tfrac{1}{2\sqrt{2}} \varepsilon t_{21}, & C_{21}(2) &= -\tfrac{1}{2\sqrt{2}} \varepsilon t_{22},\\
C_{22}(1) &= \tfrac{1}{2\sqrt{2}} \varepsilon^2 t_{21}, & C_{22}(2) &= \tfrac{1}{2\sqrt{2}} \varepsilon^2 t_{22},
\end{aligned}$$

$$\begin{aligned}
\omega\beta_{11}(1) &= \tau_{11} + 2k_5l_3 + k_9l_5, & \omega\beta_{11}(2) &= \tau_{12} + 2k_5l_3 + k_{10}l_5,\\
\omega\beta_{12}(1) &= \tau_{11} + k_5l_3 + k_6l_3 + k_9l_5, & \omega\beta_{12}(2) &= \tau_{12} + k_5l_3 + k_6l_3 + k_{10}l_5,\\
\omega\beta_{21}(1) &= \tau_{21} + k_5l_3 + k_6l_3 + k_9l_5, & \omega\beta_{21}(2) &= \tau_{22} + k_5l_3 + k_6l_3 + k_{10}l_5,\\
\omega\beta_{22}(1) &= \tau_{21} + 2k_6l_3 + k_9l_5, & \omega\beta_{22}(2) &= \tau_{22} + 2k_6l_3 + k_{10}l_5.
\end{aligned}$$

Since we only wish to keep terms of first order in ε or lower, any term in the expression for $\|\Omega(\omega)\bar{a}\|^2$ involving T_{22} or products involving T_{12} and/or T_{21} twice will be ignored. Thus, Equation 3.2 leads to the simpler expression

$$\|\Omega(\omega)\bar{a}\|^2 \cong 2|a_1|^2(1+\cos\phi)|T_{11}|^2 + 2\,\mathrm{Re}\{a_1\bar{a}_2[\overline{T}_{12}T_{11} + \overline{T}_{12}e^{i\phi}T_{11}]\} + 2\,\mathrm{Re}\{a_1\bar{a}_2[e^{-i\phi}\overline{T}_{21}T_{11} + \overline{T}_{21}T_{11}]\} .$$

When the frequency distribution g of the input is a step function, the results of Section 3 give

$$I_R = \pi \int_{-\infty}^{\infty} \|\Omega(\omega)\bar{a}\|^2 g^2(\omega)d\omega \cong \frac{\pi}{4\omega_1}\sum_{s=1}^{2}\sum_{p=1}^{2} \Big\{ \; a_1^2\, 2(1+\cos\phi)C_{11}(s)C_{11}(p)\frac{\sin[\beta_{11}(s)-\beta_{11}(p)]\omega_1}{[\beta_{11}(s)-\beta_{11}(p)]\omega_1} + 2|a_1a_2|(1+\cos\phi)C_{11}(s)C_{12}(p)\frac{\sin[\beta_{11}(s)-\beta_{12}(p)]\omega_1}{[\beta_{11}(s)-\beta_{12}(p)]\omega_1} + 2|a_1a_2|(1+\cos\phi)C_{11}(s)C_{21}(p)\frac{\sin[\beta_{11}(s)-\beta_{21}(p)]\omega_1}{[\beta_{11}(s)-\beta_{21}(p)]\omega_1} \; \Big\}, \quad (4.2)$$

where we have assumed that both a_1 and a_2 are real, thus $\alpha = 0$.

Next, we compute some of the differences $\omega_1[\beta_{ij}(s) - \beta_{ij}(p)]$ ($s,\, p = 1, 2$).

$\omega_1[\beta_{11}(1) - \beta_{11}(2)] = \tau_{11} - \tau_{12} - \Delta_5 l_5$, where $\Delta_5 = k_{10} - k_9$

$\omega_1[\beta_{11}(1) - \beta_{12}(1)] = -\Delta_3 l_3$, where $\Delta_3 = k_6 - k_5$
$\omega_1[\beta_{11}(1) - \beta_{12}(2)] = \tau_{11} - \tau_{12} - \Delta_3 l_3 - \Delta_5 l_5$
$\omega_1[\beta_{11}(2) - \beta_{12}(1)] = \tau_{12} - \tau_{11} - \Delta_3 l_3 + \Delta_5 l_5$
$\omega_1[\beta_{11}(2) - \beta_{12}(2)] = -\Delta_3 l_3$

$\omega_1[\beta_{11}(1) - \beta_{21}(1)] = \tau_{11} - \tau_{21} - \Delta_3 l_3$
$\omega_1[\beta_{11}(1) - \beta_{21}(2)] = \tau_{11} - \tau_{22} - \Delta_3 l_3 - \Delta_5 l_5$

$$\omega_1[\beta_{11}(2)-\beta_{21}(1)]=\tau_{12}-\tau_{21}-\Delta_3 l_3+\Delta_5 l_5$$
$$\omega_1[\beta_{11}(2)-\beta_{21}(2)]=\tau_{12}-\tau_{22}-\Delta_3 l_3 .$$

Let $\operatorname{sinc} x \equiv \dfrac{\sin x}{x}$ and note that $\operatorname{sinc} x = \operatorname{sinc}(-x)$. Equation 4.2 gives the output intensity as a function of the Sagnac phase shift ϕ for any value of the attenuating coefficient ε; let it be denoted by $I_\varepsilon(\phi)$

$$\begin{aligned} I_\varepsilon(\phi) \cong \frac{\pi}{16\omega_1}(1+\cos\phi)\Big\{\ & a_1^2\,[t_{11}^2+t_{12}^2-2t_{11}t_{12}\operatorname{sinc}(\Delta_5 l_5+\tau_{12}-\tau_{11})]+ \\ & |a_1 a_2|\varepsilon\,[(t_{11}^2-t_{12}^2)\operatorname{sinc}\Delta_3 l_3 + \\ & t_{11}t_{12}\big(\operatorname{sinc}(\Delta_3 l_3+\Delta_5 l_5+\tau_{12}-\tau_{11})-\operatorname{sinc}(\Delta_3 l_3-\Delta_5 l_5+\tau_{11}-\tau_{12})\big)+ \\ & t_{11}t_{21}\operatorname{sinc}(\Delta_3 l_3+\tau_{21}-\tau_{11})-t_{11}t_{22}\operatorname{sinc}(\Delta_3 l_3+\Delta_5 l_5+\tau_{22}-\tau_{11})+ \\ & -t_{12}t_{21}\operatorname{sinc}(\Delta_3 l_3-\Delta_5 l_5+\tau_{21}-\tau_{12})+t_{12}t_{22}\operatorname{sinc}(\Delta_3 l_3+\tau_{22}-\tau_{12})]\ \Big\}. \end{aligned} \tag{4.3}$$

To simplify the notation, define A and B as

$$A \equiv a_1^2\,[t_{11}^2+t_{12}^2-2t_{11}t_{12}\operatorname{sinc}(\Delta_5 l_5+\tau_{12}-\tau_{11})],$$

$$\begin{aligned} B \equiv\ & |a_1 a_2|\varepsilon\,[(t_{11}^2-t_{12}^2)\operatorname{sinc}\Delta_3 l_3 + \\ & t_{11}t_{12}\big(\operatorname{sinc}(\Delta_3 l_3+\Delta_5 l_5+\tau_{12}-\tau_{11})-\operatorname{sinc}(\Delta_3 l_3-\Delta_5 l_5+\tau_{11}-\tau_{12})\big)+ \\ & t_{11}t_{21}\operatorname{sinc}(\Delta_3 l_3+\tau_{21}-\tau_{11})-t_{11}t_{22}\operatorname{sinc}(\Delta_3 l_3+\Delta_5 l_5+\tau_{22}-\tau_{11})+ \\ & -t_{12}t_{21}\operatorname{sinc}(\Delta_3 l_3-\Delta_5 l_5+\tau_{21}-\tau_{12})+t_{12}t_{22}\operatorname{sinc}(\Delta_3 l_3+\tau_{22}-\tau_{12})] \end{aligned}$$

so that we can write $I_\varepsilon(\phi) \cong \dfrac{\pi}{16\omega_1}(1+\cos\phi)[\,A+B\,]$ and $I_o(\phi) \cong \dfrac{\pi}{16\omega_1}(1+\cos\phi)\ A$.

The Sagnac phase shift error due to polarization is given by Equation 3.13

$$\phi_\varepsilon = \sin^{-1}\left(\frac{I_\varepsilon(-\pi/2)}{I_o(-\pi/2)}-1\right) = \sin^{-1}(B/A) = \sin^{-1}\left(\frac{I_\varepsilon(\phi_s)}{I_o(\phi_s)}-1\right). \tag{4.4}$$

4.2. THE INTENSITY AS A FUNCTION OF l_5; THE OUTPUT COHERENCE

Next, we analyze the error in Equation 4.3, the difference between $I_\varepsilon(\phi)$ and $I_o(\phi)$ as a function of l_5.

$$E \equiv I_\varepsilon(\phi) - I_o(\phi) = \frac{\pi}{16\omega_1}(1+\cos\phi)B$$

The limit of E as $l_5 \to \infty$ is

$$\lim_{l_5\to\infty} E = \frac{\pi}{16\omega_1}(1+\cos\phi)\; a_1a_2\varepsilon\,[(t_{11}^2 - t_{12}^2)\,\mathrm{sinc}\,\Delta_3 l_3 + t_{11}t_{21}\,\mathrm{sinc}(\Delta_3 l_3 + \tau_{21} - \tau_{11}) + t_{12}t_{22}\,\mathrm{sinc}(\Delta_3 l_3 + \tau_{22} - \tau_{12})].$$

Assuming $\tau_{11} = \tau_{21}$ and $\tau_{12} = \tau_{22}$, this limit is zero provided

$$t_{11}^2 - t_{12}^2 + t_{11}t_{21} + t_{12}t_{22} = 0. \tag{4.5}$$

If we assume that the t_{ij} values represent a rotation through an angle θ, so that $t_{11} = \cos\theta$, $t_{12} = \sin\theta$, $t_{21} = -\sin\theta$, and $t_{22} = \cos\theta$, then Equation 4.5 implies $\cos^2\theta - \sin^2\theta = 0$, forcing $\theta = \pm 45^o, \pm 135^o, \pm 225^o$, or $\pm 315^o$. If $\theta = 45^o$ or -135^o, the error E is zero and hence $\phi_\varepsilon = 0$. If $\theta = -45^o$ or 135^o, then Equation 4.8 is satisfied and the t_{ij} values are, respectively, $(t_{11}, t_{12}, t_{21}, t_{22})$ = $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and

$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$. The error angle ϕ_ε as a function of l_5 for these two sets of values of t_{ij} is the same and is shown in Figure 1.

Examples of other sets of t_{ij} values satisfying Equation 4.5 that are not rotations include the following: $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm\alpha \cdot (1, -1, 1, 1)$ for any real value α, which generalize the previous sets of solutions. Note that these are not rotations unless $\alpha = \frac{1}{\sqrt{2}}$. The sets of values $(1, 0, -1, 1)$ and $(1, \frac{1\pm\sqrt{5}}{2}, 0, 1)$ also satisfy Equation 4.5. The error angles ϕ_ε as a function of l_5 for these sets of values of t_{ij} are shown in Figures 2, 3, and 4. Figure 5 shows the error angle as a function of l_5 for a rotation through an angle of 2 radians. Note that the error does not converge to zero as $l_5 \to \infty$.

Whenever Equation 4.5 is satisfied with $\tau_{11} = \tau_{21}$ and $\tau_{12} = \tau_{22}$, the error becomes

$$\begin{aligned} E = \tfrac{\pi}{16\omega_1}(1+\cos\phi)\, a_1 a_2 \varepsilon [\; & t_{11}t_{12}\big(\mathrm{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) - \mathrm{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{22})\big) \\ & - t_{11}t_{22}\,\mathrm{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) - t_{12}t_{21}\,\mathrm{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{22})] \end{aligned}$$

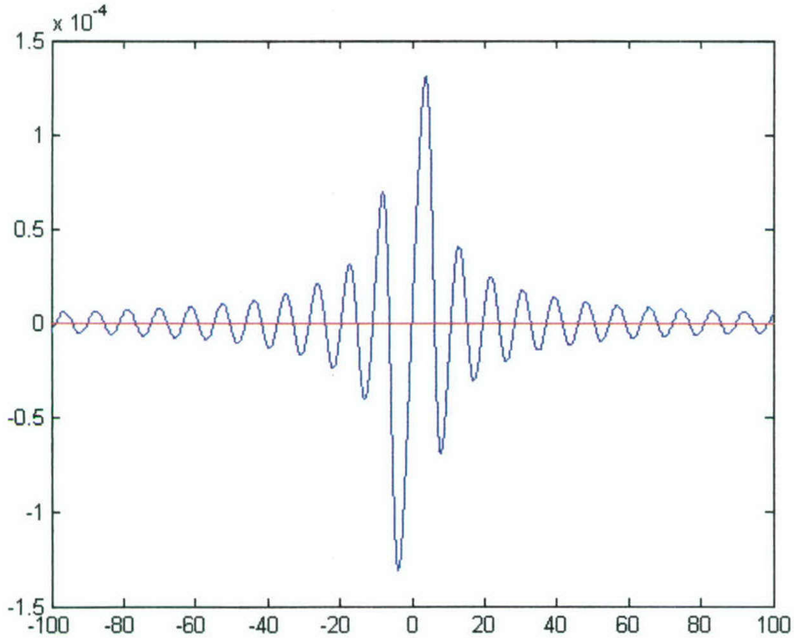

FIGURE 1. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, -45^o and 135^o Rotations.

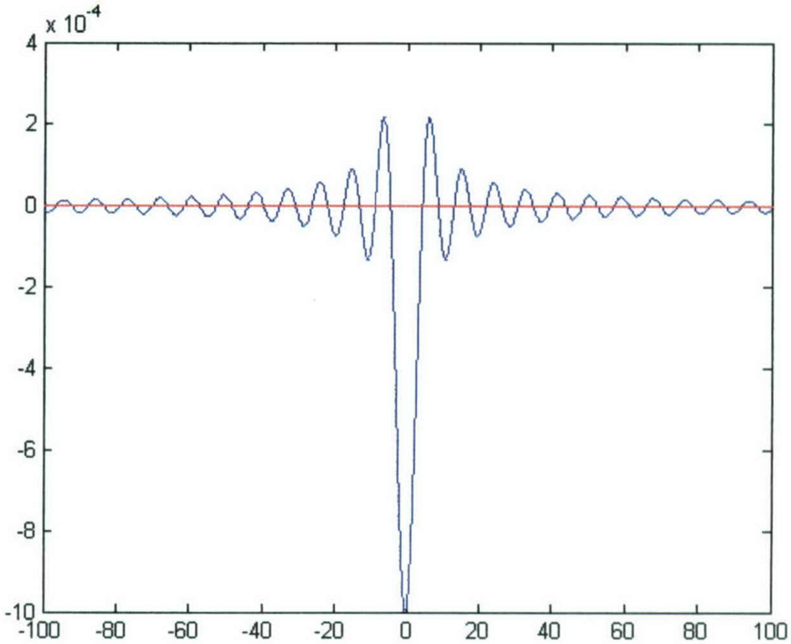

FIGURE 2. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm\,(1, 0, -1, 1)$, not a Rotation.

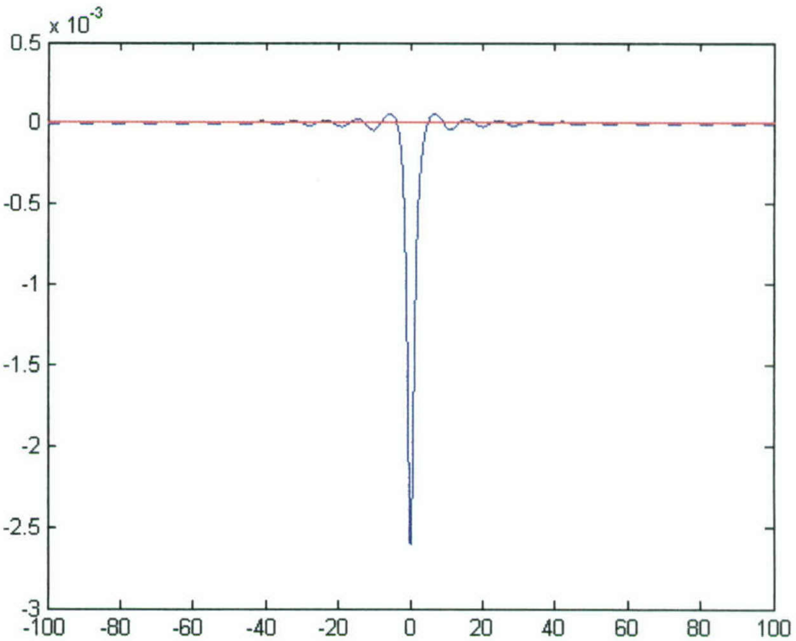

FIGURE 3. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm\,(1, \frac{1+\sqrt{5}}{2}, 0, 1)$, not a Rotation.

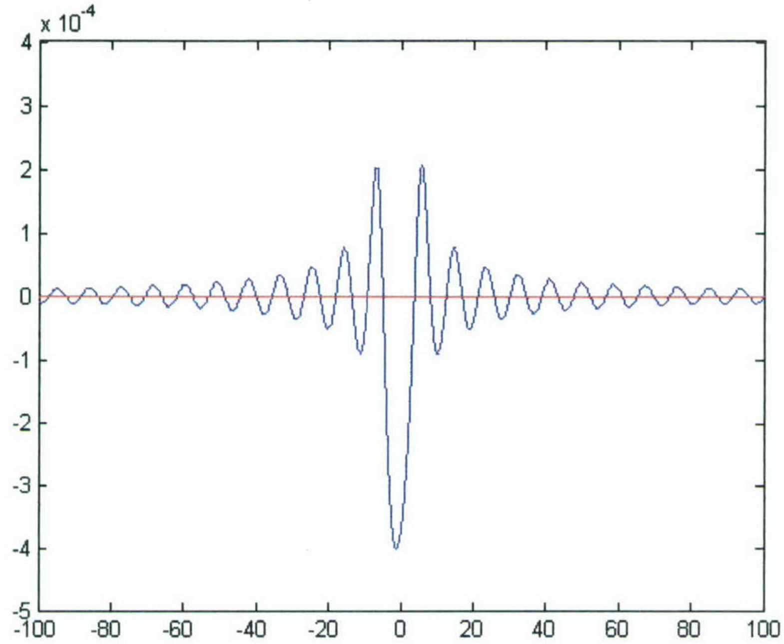

FIGURE 4. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm(1, \frac{1-\sqrt{5}}{2}, 0, 1)$, not a Rotation.

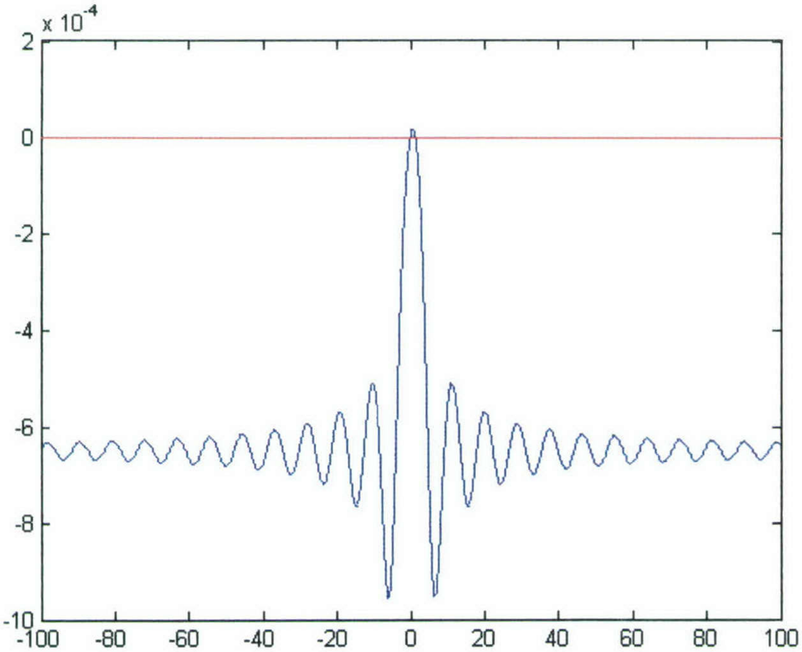

FIGURE 5. $(t_{11}, t_{12}, t_{21}, t_{22}) = \pm(\cos 2, \sin 2, -\sin 2, \cos 2)$.

4.3. MODULATION

To model the effects of modulation rewrite Equation 4.3 as

$$I_\varepsilon(\phi) \cong \frac{\pi}{16\omega_1}(A+B) + \frac{\pi}{16\omega_1}(A+B)\cos\phi .$$

We then replace $\cos\phi$ by $\cos(\phi - \phi_p \sin\omega_p t)$ and expand.

$$\begin{aligned} \cos(\phi - \phi_p \sin\omega_p t) &= \cos\phi\cos(\phi_p \sin\omega_p t) + \sin\phi\sin(\phi_p \sin\omega_p t) \\ &= \cos\phi\left[J_o(\phi_p) + 2\sum_{k=1}^{\infty} J_{2k}(\phi_p)\cos(2k\omega_p t)\right] + \sin\phi\left[2\sum_{k=0}^{\infty} J_{2k+1}(\phi_p)\sin(2k+1)\omega_p t\right]. \end{aligned}$$

If we replace $\cos\phi$ by $\cos(\phi - \phi_p \sin\omega_p t)$ in the expression for $I_\varepsilon(\phi)$, the only term with frequency $\omega_p t$ is the one with J_1, therefore the demodulated and filtered intensity will be

$$I_\varepsilon(\phi) \cong \frac{\pi}{8\omega_1} J_1(\phi_p)(A+B)\sin\phi \text{, or}$$

$$\begin{aligned} I_\varepsilon(\phi) \cong \frac{\pi}{8\omega_1} J_1(\phi_p)\sin\phi \cdot \{ \; & a_1^2\,[t_{11}^2 + t_{12}^2 - 2t_{11}t_{12}\,\mathrm{sinc}(\Delta_5 l_5 + \tau_{12} - \tau_{11})] + \\ & a_1 a_2 \varepsilon\,[(t_{11}^2 - t_{12}^2)\,\mathrm{sinc}\,\Delta_3 l_3 + \\ & t_{11}t_{12}\big(\mathrm{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{12} - \tau_{11}) - \mathrm{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{11} - \tau_{12})\big) + \\ & t_{11}t_{21}\,\mathrm{sinc}(\Delta_3 l_3 + \tau_{21} - \tau_{11}) - t_{11}t_{22}\,\mathrm{sinc}(\Delta_3 l_3 + \Delta_5 l_5 + \tau_{22} - \tau_{11}) + \\ & -t_{12}t_{21}\,\mathrm{sinc}(\Delta_3 l_3 - \Delta_5 l_5 + \tau_{21} - \tau_{12}) + t_{12}t_{22}\,\mathrm{sinc}(\Delta_3 l_3 + \tau_{22} - \tau_{12})] \; \}. \end{aligned}$$

To obtain an expression for the rotation angle error ϕ_ε when the output of the gyro is demodulated and filtered, write $I_\varepsilon(\phi)$ as $I_\varepsilon(\phi) \cong \frac{\pi}{8\omega_1} J_1(\phi_p)[A+B]\sin\phi$ and $I_o(\phi) \cong \frac{\pi}{8\omega_1} J_1(\phi_p) A \sin\phi$.

If we set $I_\varepsilon(\phi) = I_o(\phi + \phi_\varepsilon)$ and solve for ϕ_ε we get

$$\frac{\pi}{8\omega_1} J_1(\phi_p)[A+B]\sin\phi = \frac{\pi}{8\omega_1} J_1(\phi_p) A \sin(\phi+\phi_\varepsilon)$$

$$\Rightarrow \qquad \sin\phi \cdot [A+B] = [\sin\phi\cos\phi_\varepsilon + \cos\phi\sin\phi_\varepsilon]\ A$$

$$\phi_\varepsilon \cong 0 \ \Rightarrow \qquad \sin\phi \cdot [A+B] \cong [\sin\phi + \sin\phi_\varepsilon \cos\phi]\ A$$

$$\Rightarrow \qquad \phi_\varepsilon = \sin^{-1}\left(\frac{B\sin\phi}{A\cos\phi}\right) \cong \frac{B\sin\phi}{A\cos\phi} \quad \text{for all } \phi \text{ such that } \cos\phi \neq 0\,.$$

In particular, if $\phi = 45^o$ then $\phi_\varepsilon = \sin^{-1}(B/A) = \sin^{-1}\left(\frac{I_\varepsilon(\phi)}{I_o(\phi)} - 1\right)$ as in Equation 4.4.

5. A MORE GENERAL IFOG MODEL

Our second example is a more general gyroscope. The clockwise loop in the second example consists of an IOC connected to a length l_2 of PM fiber at an angle α_1 which represents a miss-alignment and is therefore nominally small, however, in our analysis and simulation it can have any value. The PM fiber is connected to a second IOC at an angle α_2 representing a miss-alignment but may have arbitrary value. The second IOC is

connected to a length l_4 of PM fiber at an angle α_3 representing a miss-alignment of a zero or a 90 degree splicing. This PM fiber is connected to a length l_5 of PM fiber at a 45 degree angle plus an error angle β_1 representing a miss-alignment. The PM fiber is connected to the coil which is then connected to a length l_6 of PM fiber that is connected at a $(45^{\circ} + \beta_2)$ angle to a length l_7 of PM fiber. The last piece of fiber is connected to the second IOC at an angle α_4 representing a miss-alignment of a zero or a 90 degree splicing. The path returns back from the second IOC to the first IOC through the same forward path connections. The clockwise loop is depicted symbolically in Figure 6.

$$\begin{bmatrix} \text{CHIP - 1} \\ k_1, k_2, l_1 \\ \varepsilon_1 \end{bmatrix} \leftrightarrow [0^0 \pm \alpha_1] \leftrightarrow [PMF(k_{3,4}, l_2)] \leftrightarrow [0^0 \pm \alpha_2] \leftrightarrow \begin{bmatrix} \text{CHIP - 2} \\ k_5, k_6, l_3 \\ \varepsilon_2 \end{bmatrix} \begin{matrix} \leftrightarrow \\ \\ \leftrightarrow \end{matrix}$$

$$\begin{matrix} \leftrightarrow \\ \\ \leftrightarrow \end{matrix} \begin{bmatrix} \text{CHIP - 2} \\ k_5, k_6, l_3 \\ \varepsilon_2 \end{bmatrix} \begin{matrix} \leftrightarrow [0^0 \, or \, 90^0 \pm \alpha_3] \leftrightarrow [PMF(k_{7,8}, l_4)] \leftrightarrow [45^0 \pm \beta_1] \leftrightarrow [PMF(k_{9,10}, l_5)] \leftrightarrow \\ \\ \leftrightarrow [0^0 \, or \, 90^0 \pm \alpha_4] \leftrightarrow [PMF(k_{13}, k_{14}, l_7)] \leftrightarrow [45^0 \pm \beta_2] \leftrightarrow [PMF(k_{11,12}, l_6)] \leftrightarrow \end{matrix} \left(\left(\text{COIL} \right) \right)$$

FIGURE 6. Interferometric Fiber Optic Gyroscope.

5.1. THE JONES MATRICES OF THE COMPONENTS

The Jones matrices of the gyro components are as follows. The forward path of the two IOCs have Jones matrices M_1 and M_2 given by

$$M_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_1 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_1 l_1} & 0 \\ 0 & e^{ik_2 l_1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{ik_1 l_1} & 0 \\ 0 & \varepsilon_1 e^{ik_2 l_1} \end{bmatrix} = \frac{e^{ik_1 l_1}}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_1 e^{i(k_2 - k_1) l_1} \end{bmatrix}$$

and

$$M_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & e^{ik_6 l_3} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{ik_5 l_3} & 0 \\ 0 & \varepsilon_2 e^{ik_6 l_3} \end{bmatrix} = \frac{e^{ik_5 l_3}}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_2 e^{i(k_6 - k_5) l_3} \end{bmatrix},$$

where $k_s = \frac{\omega}{c} n_s$ for all s and n_s is the index of refraction along the respective path. The return path of the IOCs have Jones matrices M_{1c} and M_{2c} given by

$$M_{1c} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_1 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_1 l_9} & 0 \\ 0 & e^{ik_2 l_9} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{ik_1 l_9} & 0 \\ 0 & \varepsilon_1 e^{ik_2 l_9} \end{bmatrix} = \frac{e^{ik_1 l_9}}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_1 e^{i(k_2 - k_1) l_9} \end{bmatrix}$$

and

$$M_{2c} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_2 \end{bmatrix} \cdot \begin{bmatrix} e^{ik_5 l_8} & 0 \\ 0 & e^{ik_6 l_8} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{ik_5 l_8} & 0 \\ 0 & \varepsilon_2 e^{ik_6 l_8} \end{bmatrix} = \frac{e^{ik_5 l_8}}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & \varepsilon_2 e^{i(k_6 - k_5) l_8} \end{bmatrix},$$

The lengths of PM fiber have Jones matrices $U_1, U_2, \ldots, U_5$ given by

$$U_1 = \begin{bmatrix} e^{ik_3 l_2} & 0 \\ 0 & e^{ik_4 l_2} \end{bmatrix}, \quad U_2 = \begin{bmatrix} e^{ik_7 l_4} & 0 \\ 0 & e^{ik_8 l_4} \end{bmatrix}, \quad U_3 = \begin{bmatrix} e^{ik_9 l_5} & 0 \\ 0 & e^{ik_{10} l_5} \end{bmatrix},$$

$$U_4 = \begin{bmatrix} e^{ik_{11} l_6} & 0 \\ 0 & e^{ik_{12} l_6} \end{bmatrix}, \quad U_5 = \begin{bmatrix} e^{ik_{13} l_7} & 0 \\ 0 & e^{ik_{14} l_7} \end{bmatrix}.$$

These matrices can be written in the form $U = e^{ikl}\begin{bmatrix} 1 & 0 \\ 0 & e^{i(k' - k)l} \end{bmatrix}$ and the factor e^{ikl} can be ignored when computing the output intensity.

The rotations are represented by R_1, R_2, ... , R_6, where

$$R_1 = \begin{bmatrix} \cos\alpha_1 & \sin\alpha_1 \\ -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}, \quad R_2 = \begin{bmatrix} \cos\alpha_2 & \sin\alpha_2 \\ -\sin\alpha_2 & \cos\alpha_2 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 \\ -\sin\alpha_3 & \cos\alpha_3 \end{bmatrix} \text{ or } \begin{bmatrix} -\sin\alpha_3 & \cos\alpha_3 \\ -\cos\alpha_3 & -\sin\alpha_3 \end{bmatrix},$$

a rotation through and angle ($0^o + \alpha_3$) or ($90^o + \alpha_3$).

$$R_4 = \begin{bmatrix} \cos(45^0 + \beta_1) & \sin(45^0 + \beta_1) \\ -\sin(45^0 + \beta_1) & \cos(45^0 + \beta_1) \end{bmatrix}, \quad R_5 = \begin{bmatrix} \cos(45^0 + \beta_2) & \sin(45^0 + \beta_2) \\ -\sin(45^0 + \beta_2) & \cos(45^0 + \beta_2) \end{bmatrix},$$

and

$$R_6 = \begin{bmatrix} \cos\alpha_4 & \sin\alpha_4 \\ -\sin\alpha_4 & \cos\alpha_4 \end{bmatrix} \text{ or } \begin{bmatrix} -\sin\alpha_4 & \cos\alpha_4 \\ -\cos\alpha_4 & -\sin\alpha_4 \end{bmatrix},$$

a rotation through and angle ($0^o + \alpha_4$) or ($90^o + \alpha_4$).

We use a general 2x2 complex matrix A for the Jones matrix of the coil,

$$A = \begin{bmatrix} t_{11}e^{i\tau_{11}} & t_{12}e^{i\tau_{12}} \\ t_{21}e^{i\tau_{21}} & t_{22}e^{i\tau_{22}} \end{bmatrix}.$$

5.2. THE CLOCKWISE AND COUNTER-CLOCKWISE LOOPS

Let $C_1 = R_2 \cdot U_1 \cdot R_1$, $C_2 = U_3 \cdot R_4 \cdot U_2 \cdot R_3$, and $C_3 = R_6 \cdot U_5 \cdot R_5 \cdot U_4$. Then, the clockwise-path is given by

$$T = [(-i) \cdot M_1^T \cdot C_1^T \cdot (-i) \cdot M_{2c}^T] \cdot [C_3 \cdot A \cdot C_2] \cdot [M_2 \cdot C_1 \cdot M_1] = - T_1^T \cdot T_2 \cdot T_1, \tag{5.1}$$

where we have set $T_1 = M_2 \cdot C_1 \cdot M_1$, and $T_2 = C_3 \cdot T \cdot C_2$.

Note that $M_1^T = M_1$ and $M_{2c}^T = M_{2c}$.

The counter clockwise-path T_c is given by

$$T_c = [(-i) \cdot M_1^T \cdot C_1^T \cdot M_2^T] \cdot [C_2^T \cdot A^T \cdot C_3^T] \cdot [(-i) \cdot M_{2c} \cdot C_1 \cdot M_1]. \tag{5.2}$$

Comparing Equations 5.1 and 5.2, we conclude that $T_c = T^T$.

The four entries T_{11}, T_{12}, T_{21}, and T_{22} of the clockwise path matrix T are computed in Appendix A. The end result of that computation are the four arrays C_{11}, C_{12}, C_{21}, C_{22} of 256 coefficients and four arrays Λ_{11}, Λ_{12}, Λ_{21}, Λ_{22}, of 256 sets of indices needed to define the four entries of the clockwise IFOG Jones matrix as defined by Equation 3.3.

We wrote Matlab programs to compute the output intensities given by Equations 3.14, 3.15, and 3.19 and to compute the Sagnac error given by Equation 3.13 for the IFOG model presented here for Gaussian, rectangular, and triangular frequency distribution functions. These programs are described in Appendix B where we include their listings. The plots in Figures 7, 8, and 9 were produced using these programs.

Figures 7, 8, and 9 show the output of the function **SagnacBias** for a Gaussian, step, and triangular distribution function, respectively. The horizontal axis shows the values of

the length l_7 and the vertical axis shows the values of the length l_4 in meters. The color bars show the value of the bias angle in radians for the different colors.

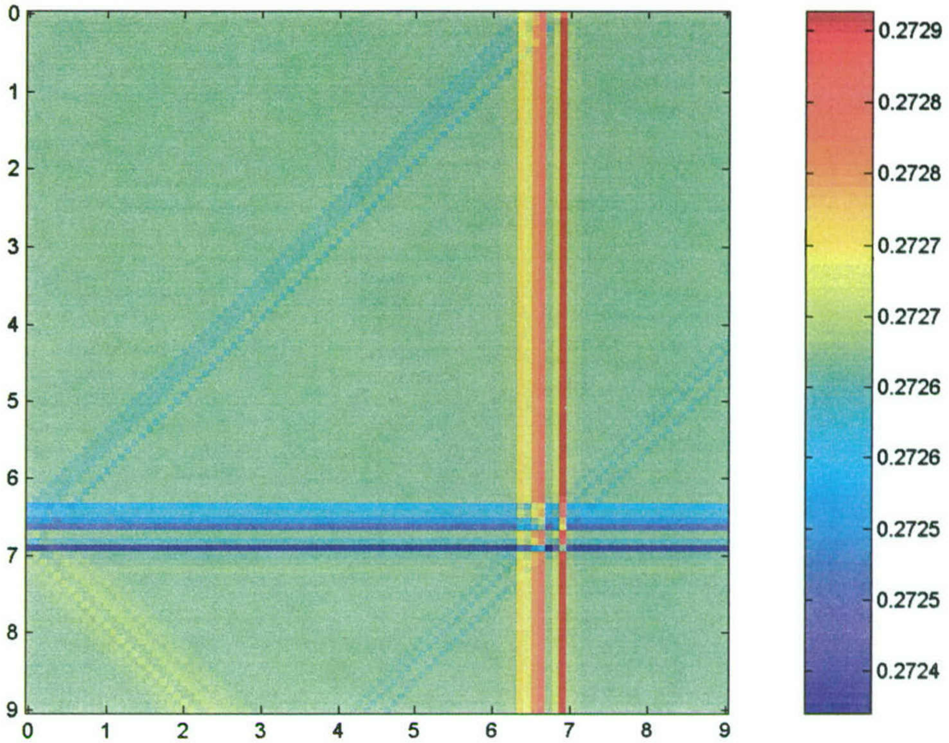

FIGURE 7. Bias Error - Gaussian Distribution.

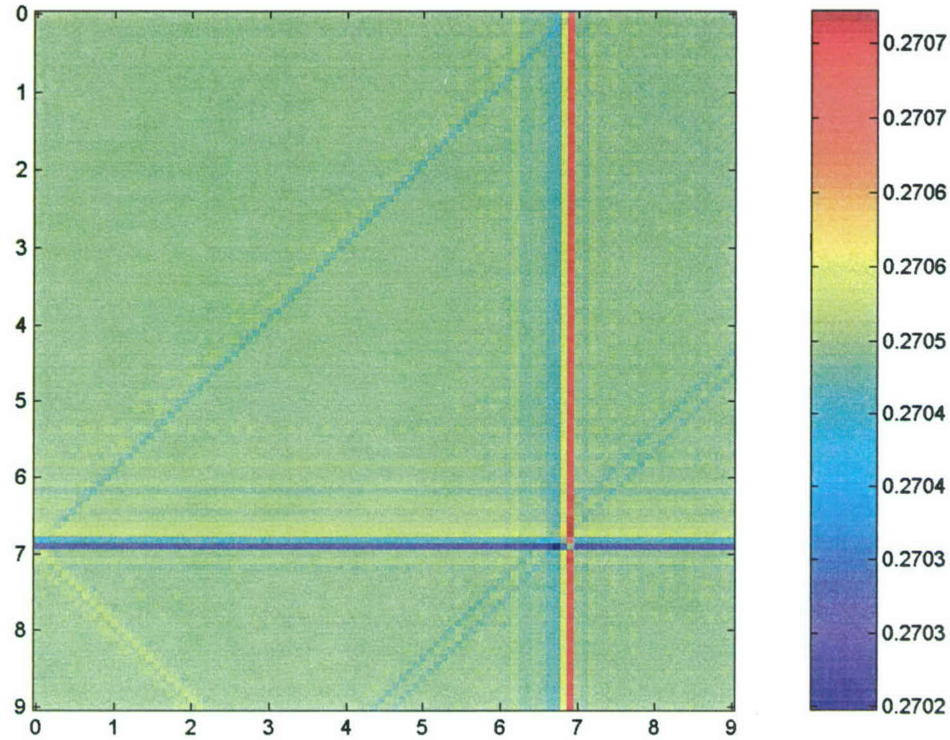

FIGURE 8. Bias Error - Step Distribution.

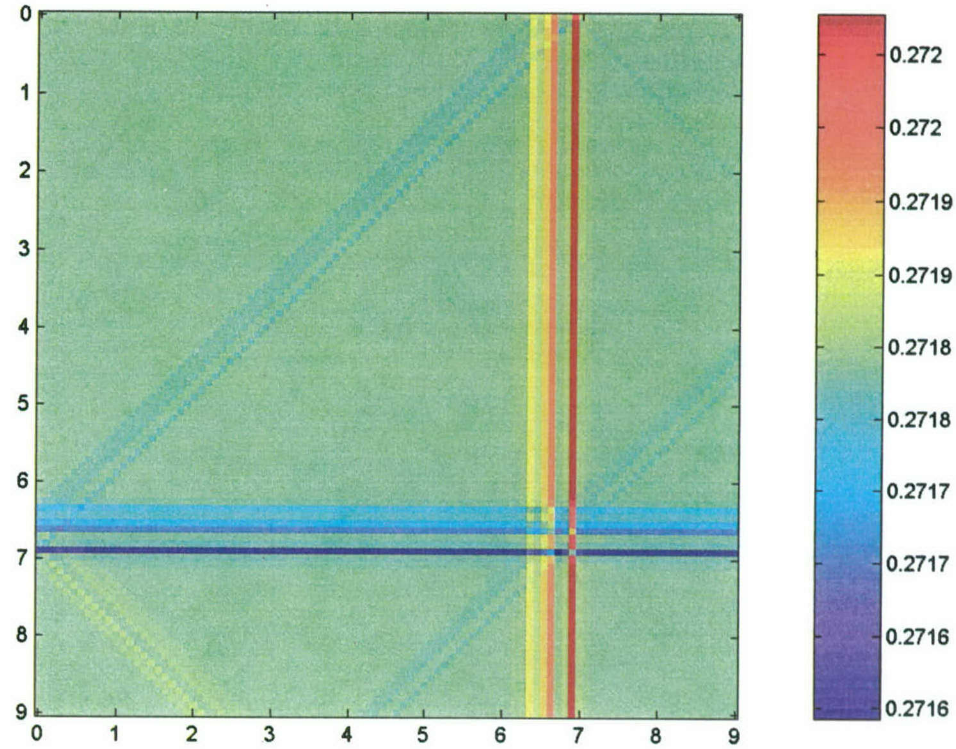

FIGURE 9. Bias Error - Triangular Distribution.

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Appendix A

THE CLOCKWISE-PATH MATRIX T

In this Appendix we compute the matrix T defined by Equation 5.1. As before, we write T as $T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$. Since there are 8 rotations along the clockwise path, each T_{ij} is a sum of $2^8 = 256$ terms. They have the form (see Example 1, Section 4)

$$T_{ij}(\omega) = \sum_{s=1}^{256} C_{ij}(s) \exp(i \sum_{r \in \Lambda_{ij}(s)} \phi_r),$$

where each C_{ij} is a 256-array of coefficients, each $\Lambda_{ij}(s)$ is a set of 11 indices, (s = 1, 2, ..., 256; i, j = 1, 2), and $\phi_1 = k_1 l_1$, $\phi_2 = k_2 l_1$, $\phi_3 = k_3 l_2$, $\phi_4 = k_4 l_2$, ... , $\phi_r = k_r l_{\frac{r+1}{2}}$ for r odd, $\phi_r = k_r l_{\frac{r}{2}}$ for r even, where $k_r = n_r \omega / c$ for r = 1, 2, 3, ... 14, and r = 19, 20, 21, and 22.

Let θ_r denote the angle in rotation R_r and let $s_r = \sin\theta_r$ and $c_r = \cos\theta_r$ for r = 1, 2, ... , 6. The four taus, τ_{11}, τ_{12}, τ_{21}, and τ_{22} representing the coil phase shifts will receive the indices 15, 16, 17, and 18, respectively; that is $\tau_{11} = \phi_{15}$, $\tau_{12} = \phi_{16}$, $\tau_{21} = \phi_{17}$, and $\tau_{22} = \phi_{18}$.

An exponential of the form $e^{i(\phi_{r_1} + \phi_{r_2} + \ldots + \phi_{r_s})}$ will be represented by a row of indices $[r_1, r_r, \ldots, r_s]$, thus [2, 3, 5] represents $e^{i(\phi_2 + \phi_3 + \phi_5)}$ and a product $e^{i(\phi_2 + \phi_3 + \phi_5)}\, e^{i\phi_6}$ will be represented by [2,3,5][6] = [2,3,5,6] = $e^{i(\phi_2 + \phi_3 + \phi_5 + \phi_6)}$.

The product of the first r Jones matrices will be denoted by $P_r = \begin{bmatrix} P_{r,11} & P_{r,12} \\ P_{r,21} & P_{r,22} \end{bmatrix}$.

$$P_2 = \begin{bmatrix} c_1 & s_1 \\ -s_1 & c_1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi_1} & 0 \\ 0 & \varepsilon_1 e^{i\phi_2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_1 e^{i\phi_1} & s_1\varepsilon_1 e^{i\phi_2} \\ -s_1 e^{i\phi_1} & c_1\varepsilon_1 e^{i\phi_2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[1] & s_1\varepsilon_1[2] \\ (-s_1)[1] & c_1\varepsilon_1[2] \end{bmatrix}.$$

$$P_4 = R_2U_1P_2 = \begin{bmatrix} c_2 & s_2 \\ -s_2 & c_2 \end{bmatrix} \begin{bmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[1] & s_1\varepsilon_1[2] \\ (-s_1)[1] & c_1\varepsilon_1[2] \end{bmatrix} =$$

$$= \begin{bmatrix} c_2[3] & s_2[4] \\ (-s_2)[3] & c_2[4] \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[1] & s_1\varepsilon_1[2] \\ (-s_1)[1] & c_1\varepsilon_1[2] \end{bmatrix} =$$

$$P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} c_2c_1[1,3] + s_2(-s_1)[1,4] & c_2s_1\varepsilon_1[2,3] + s_2c_1\varepsilon_1[2,4] \\ (-s_2)c_1[1,3] + c_2(-s_1)[1,4] & (-s_2)s_1\varepsilon_1[2,3] + c_2c_1\varepsilon_1[2,4] \end{bmatrix}.$$

Thus,

$P_{4,11} = \frac{1}{\sqrt{2}} \{ c_2c_1[1,3] + s_2(-s_1)[1,4] \}$, $P_{4,12} = \frac{1}{\sqrt{2}} \{ c_2s_1\varepsilon_1[2,3] + s_2c_1\varepsilon_1[2,4] \}$,
$P_{4,21} = \frac{1}{\sqrt{2}} \{ (-s_2)c_1[1,3] + c_2(-s_1)[1,4] \}$, $P_{4,22} = \frac{1}{\sqrt{2}} \{ (-s_2)s_1\varepsilon_1[2,3] + c_2c_1\varepsilon_1 [2,4] \}$,

where the red symbols indicate small numbers.

$$P_6 = R_3M_2P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} c_3 & s_3 \\ -s_3 & c_3 \end{bmatrix} \begin{bmatrix} e^{i\phi_5} & 0 \\ 0 & \varepsilon_2 e^{i\phi_6} \end{bmatrix} \cdot P_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} c_3[5] & s_3\varepsilon_2[6] \\ (-s_3)[5] & c_3\varepsilon_2[6] \end{bmatrix} \cdot P_4.$$

Thus,

$P_{6,11} = \frac{1}{\sqrt{2}} c_3 [5] P_{4,11} + \frac{1}{\sqrt{2}} s_3\varepsilon_2 [6] P_{4,21}$
$P_{6,12} = \frac{1}{\sqrt{2}} c_3 [5] P_{4,12} + \frac{1}{\sqrt{2}} s_3\varepsilon_2 [6] P_{4,22}$
$P_{6,21} = \frac{1}{\sqrt{2}} (-s_3)[5] P_{4,11} + \frac{1}{\sqrt{2}} c_3\varepsilon_2 [6] P_{4,21}$
$P_{6,22} = \frac{1}{\sqrt{2}} (-s_3)[5] P_{4,12} + \frac{1}{\sqrt{2}} c_3\varepsilon_2 [6] P_{4,22}$

$P_{6,11} = \frac{1}{2} \{ c_3c_2c_1[1,3,5] + c_3s_2(-s_1)[1,4,5] + s_3\varepsilon_2 (-s_2)c_1[1,3,6] + s_3\varepsilon_2 c_2(-s_1)[1,4,6] \}$
$P_{6,12} = \frac{1}{2} \{ c_3c_2 s_1\varepsilon_1[2,3,5] + c_3 s_2c_1\varepsilon_1[2,4,5] + s_3\varepsilon_2 (-s_2)s_1\varepsilon_1[2,3,6] + s_3\varepsilon_2 c_2c_1\varepsilon_1[2,4,6] \}$
$P_{6,21} = \frac{1}{2} \{(-s_3)c_2c_1[1,3,5] + (-s_3)s_2(-s_1)[1,4,5] + c_3\varepsilon_2(-s_2)c_1[1,3,6] + c_3\varepsilon_2c_2(-s_1)[1,4,6] \}$
$P_{6,22} = \frac{1}{2} \{(-s_3) c_2s_1\varepsilon_1[2,3,5] + (-s_3) s_2c_1\varepsilon_1[2,4,5] + c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6] +$

$c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6]$ },

where the blue symbols indicate numbers that could be small.

$$P_8 = R_4U_2P_6 = \begin{bmatrix} c_4 & s_4 \\ -s_4 & c_4 \end{bmatrix}\begin{bmatrix} e^{i\phi_7} & 0 \\ 0 & e^{i\phi_8} \end{bmatrix} \cdot P_6 = \begin{bmatrix} c_4[7] & s_4[8] \\ (-s_4)[7] & c_4[8] \end{bmatrix} \cdot P_6$$

Thus,

$$P_{8,11} = c_4[7]\, P_{6,11} + s_4[8]\, P_{6,21}$$
$$P_{8,12} = c_4[7]\, P_{6,12} + s_4[8]\, P_{6,22}$$
$$P_{8,21} = (-s_4)[7]\, P_{6,11} + c_4[8]\, P_{6,21}$$
$$P_{8,22} = (-s_4)[7]\, P_{6,12} + c_4[8]\, P_{6,22}$$

$$P_{8,11} = \tfrac{1}{2}\ \{\ c_4c_3c_2c_1[1,3,5,7] + c_4c_3s_2(-s_1)[1,4,5,7] + c_4s_3\varepsilon_2\ (-s_2)c_1[1,3,6,7] + c_4s_3\varepsilon_2\ c_2(-s_1)[1,4,6,7] + s_4(-s_3)c_2c_1[1,3,5,8] + s_4(-s_3)s_2(-s_1)[1,4,5,8] + s_4c_3\varepsilon_2(-s_2)c_1[1,3,6,8] + s_4c_3\varepsilon_2c_2(-s_1)[1,4,6,8]\ \}$$

$$P_{8,12} = \tfrac{1}{2}\ \{\ c_4c_3c_2\ s_1\varepsilon_1[2,3,5,7] + c_4c_3\ s_2c_1\varepsilon_1[2,4,5,7] + c_4s_3\varepsilon_2\ (-s_2)s_1\varepsilon_1[2,3,6,7] + c_4s_3\varepsilon_2\ c_2c_1\varepsilon_1[2,4,6,7] + s_4(-s_3)\ c_2s_1\varepsilon_1[2,3,5,8] + s_4(-s_3)\ s_2c_1\varepsilon_1[2,4,5,8] + s_4c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6,8] + s_4c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,8]\ \}$$

$$P_{8,21} = \tfrac{1}{2}\ \{\ (-s_4)c_3c_2c_1[1,3,5,7] + (-s_4)c_3s_2(-s_1)[1,4,5,7] + (-s_4)s_3\varepsilon_2\ (-s_2)c_1[1,3,6,7] + (-s_4)s_3\varepsilon_2\ c_2(-s_1)[1,4,6,7] + c_4(-s_3)c_2c_1[1,3,5,8] + c_4(-s_3)s_2(-s_1)[1,4,5,8] + c_4c_3\varepsilon_2(-s_2)c_1[1,3,6,8] + c_4c_3\varepsilon_2c_2(-s_1)[1,4,6,8]\ \}$$

$$P_{8,22} = \tfrac{1}{2}\ \{\ (-s_4)c_3c_2\ s_1\varepsilon_1[2,3,5,7] + (-s_4)c_3\ s_2c_1\varepsilon_1[2,4,5,7] + (-s_4)s_3\varepsilon_2\ (-s_2)s_1\varepsilon_1[2,3,6,7] + (-s_4)s_3\varepsilon_2\ c_2c_1\varepsilon_1[2,4,6,7] + c_4(-s_3)\ c_2s_1\varepsilon_1[2,3,5,8] + c_4(-s_3)\ s_2c_1\varepsilon_1[2,4,5,8] + c_4c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6,8] + c_4c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,8]\ \}.$$

$$P_{11} = U_4TU_3P_8 = \begin{bmatrix} e^{i\phi_{11}} & 0 \\ 0 & e^{i\phi_{12}} \end{bmatrix} \cdot \begin{bmatrix} t_{11}e^{i\tau_{11}} & t_{12}e^{i\tau_{12}} \\ t_{21}e^{i\tau_{21}} & t_{22}e^{i\tau_{22}} \end{bmatrix}\begin{bmatrix} e^{i\phi_9} & 0 \\ 0 & e^{i\phi_{10}} \end{bmatrix} \cdot P_8$$

$$P_{11} = \begin{bmatrix} t_{11}[9,15,11] & t_{12}[10,16,11] \\ t_{21}[9,17,12] & t_{22}[10,18,12] \end{bmatrix} \cdot P_8 .$$

Thus,

$$P_{11,11} = t_{11}\,[9,15,11]\,P_{8,11} + t_{12}\,[10,16,11]\,P_{8,21}$$
$$P_{11,12} = t_{11}\,[9,15,11]\,P_{8,12} + t_{12}\,[10,16,11]\,P_{8,22}$$
$$P_{11,21} = t_{21}\,[9,17,12]\,P_{8,11} + t_{22}\,[10,18,12]\,P_{8,21}$$
$$P_{11,22} = t_{21}\,[9,17,12]\,P_{8,12} + t_{22}\,[10,18,12]\,P_{8,22}$$

$$\begin{aligned} P_{11,11} = \tfrac{1}{2} \{\, & t_{11} \{ c_4c_3c_2c_1[1,3,5,7,9,15,11] + c_4c_3s_2(-s_1)[1,4,5,7,9,15,11] + \\ & c_4s_3\varepsilon_2 (-s_2)c_1[1,3,6,7,9,15,11] + c_4s_3\varepsilon_2\, c_2(-s_1)[1,4,6,7,9,15,11] + \\ & s_4(-s_3)c_2c_1[1,3,5,8,9,15,11] + s_4(-s_3)s_2(-s_1)[1,4,5,8,9,15,11] + \\ & s_4c_3\varepsilon_2(-s_2)c_1[1,3,6,8,9,15,11] + s_4c_3\varepsilon_2c_2(-s_1)[1,4,6,8,9,15,11] \} + \\ & t_{12} \{ (-s_4)c_3c_2c_1[1,3,5,7,10,16,11] + (-s_4)c_3s_2(-s_1)[1,4,5,7,10,16,11] + \\ & (-s_4)s_3\varepsilon_2 (-s_2)c_1[1,3,6,7,10,16,11] + (-s_4)s_3\varepsilon_2\, c_2(-s_1)[1,4,6,7,10,16,11] + \\ & c_4(-s_3)c_2c_1[1,3,5,8,10,16,11] + c_4(-s_3)s_2(-s_1)[1,4,5,8,10,16,11] + \\ & c_4c_3\varepsilon_2(-s_2)c_1[1,3,6,8,10,16,11] + c_4c_3\varepsilon_2c_2(-s_1)[1,4,6,8,10,16,11] \} \,\} \end{aligned}$$

$$\begin{aligned} P_{11,12} = \tfrac{1}{2} \{\, & t_{11} \{ c_4c_3c_2\, s_1\varepsilon_1[2,3,5,7,9,15,11] + c_4c_3\, s_2c_1\varepsilon_1[2,4,5,7,9,15,11] + \\ & c_4s_3\varepsilon_2 (-s_2)s_1\varepsilon_1[2,3,6,7,9,15,11] + c_4s_3\varepsilon_2\, c_2c_1\varepsilon_1[2,4,6,7,9,15,11] + \\ & s_4(-s_3)\, c_2s_1\varepsilon_1[2,3,5,8,9,15,11] + s_4(-s_3)\, s_2c_1\varepsilon_1[2,4,5,8,9,15,11] + \\ & s_4c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6,8,9,15,11] + s_4c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,8,9,15,11] \} + \\ & t_{12} \{ (-s_4)c_3c_2\, s_1\varepsilon_1[2,3,5,7,10,16,11] + (-s_4)c_3s_2c_1\varepsilon_1[2,4,5,7,10,16,11] + \\ & (-s_4)s_3\varepsilon_2 (-s_2)s_1\varepsilon_1[2,3,6,7,10,16,11] + (-s_4)s_3\varepsilon_2\, c_2c_1\varepsilon_1[2,4,6,7,10,16,11] + \\ & c_4(-s_3)\, c_2s_1\varepsilon_1[2,3,5,8,10,16,11] + c_4(-s_3)\, s_2c_1\varepsilon_1[2,4,5,8,10,16,11] + \\ & c_4c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6,8,10,16,11] + c_4c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,8,10,16,11] \} \,\} \end{aligned}$$

$$\begin{aligned} P_{11,21} = \tfrac{1}{2} \{\, & t_{21} \{ c_4c_3c_2c_1[1,3,5,7,9,17,12] + c_4c_3s_2(-s_1)[1,4,5,7,9,17,12] + \\ & c_4s_3\varepsilon_2 (-s_2)c_1[1,3,6,7,9,17,12] + c_4s_3\varepsilon_2\, c_2(-s_1)[1,4,6,7,9,17,12] + \\ & s_4(-s_3)c_2c_1[1,3,5,8,9,17,12] + s_4(-s_3)s_2(-s_1)[1,4,5,8,9,17,12] + \\ & s_4c_3\varepsilon_2(-s_2)c_1[1,3,6,8,9,17,12] + s_4c_3\varepsilon_2c_2(-s_1)[1,4,6,8,9,17,12] \} + \\ & t_{22} \{ (-s_4)c_3c_2c_1[1,3,5,7,10,18,12] + (-s_4)c_3s_2(-s_1)[1,4,5,7,10,18,12] + \\ & (-s_4)s_3\varepsilon_2 (-s_2)c_1[1,3,6,7,10,18,12] + (-s_4)s_3\varepsilon_2\, c_2(-s_1)[1,4,6,7,10,18,12] + \\ & c_4(-s_3)c_2c_1[1,3,5,8,10,18,12] + c_4(-s_3)s_2(-s_1)[1,4,5,8,10,18,12] + \\ & c_4c_3\varepsilon_2(-s_2)c_1[1,3,6,8,10,18,12] + c_4c_3\varepsilon_2c_2(-s_1)[1,4,6,8,10,18,12] \} \,\} \end{aligned}$$

$$
\begin{aligned}
P_{11,22} = \tfrac{1}{2}\{\, & t_{21}\,\{\, c_4c_3c_2\, s_1\varepsilon_1[2,3,5,7,9,17,12] + c_4c_3s_2c_1\varepsilon_1[2,4,5,7,9,17,12] + \\
& c_4s_3\varepsilon_2\,(-s_2)s_1\varepsilon_1[2,3,6,7,9,17,12] + c_4s_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,7,9,17,12] + \\
& s_4(-s_3)\,c_2s_1\varepsilon_1[2,3,5,8,9,17,12] + s_4(-s_3)s_2c_1\varepsilon_1[2,4,5,8,9,17,12] + \\
& s_4c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6,8,9,17,12] + s_4c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,8,9,17,12]\,\} + \\
& t_{22}\,\{\, (-s_4)c_3c_2\, s_1\varepsilon_1[2,3,5,7,10,18,12] + (-s_4)c_3s_2c_1\varepsilon_1[2,4,5,7,10,18,12] + \\
& (-s_4)s_3\varepsilon_2\,(-s_2)s_1\varepsilon_1[2,3,6,7,10,18,12] + (-s_4)s_3\varepsilon_2\,c_2c_1\varepsilon_1[2,4,6,7,10,18,12] + \\
& c_4(-s_3)\,c_2s_1\varepsilon_1[2,3,5,8,10,18,12] + c_4(-s_3)s_2c_1\varepsilon_1[2,4,5,8,10,18,12] + \\
& c_4c_3\varepsilon_2(-s_2)s_1\varepsilon_1[2,3,6,8,10,18,12] + c_4c_3\varepsilon_2c_2c_1\varepsilon_1[2,4,6,8,10,18,12]\,\}\,\}
\end{aligned}
$$

(16 terms each $P_{11,ij}$)

$$P_{13} = U_5R_5P_{11} = \begin{bmatrix} e^{i\phi_{13}} & 0 \\ 0 & e^{i\phi_{14}} \end{bmatrix}\begin{bmatrix} c_5 & s_5 \\ -s_5 & c_5 \end{bmatrix}P_{11} = \begin{bmatrix} c_5[13] & s_5[13] \\ -s_5[14] & c_5[14] \end{bmatrix}P_{11}$$

Thus,

$$
\begin{aligned}
P_{13,11} &= [13]\{\, c_5P_{11,11} + s_5P_{11,21} \,\} \\
P_{13,12} &= [13]\{\, c_5P_{11,12} + s_5P_{11,22} \,\} \\
P_{13,21} &= [14]\{\, -s_5P_{11,11} + c_5P_{11,21} \,\} \\
P_{13,22} &= [14]\{\, -s_5P_{11,12} + c_5P_{11,22} \,\}.
\end{aligned}
$$

Remark. According to Equations 3.3 and 3.4, to define the four entries T_{11}, T_{12}, T_{21}, T_{22}, of the transfer function T and to compute the output intensities for various g functions we need the four arrays of coefficients C_{11}, C_{12}, C_{21}, C_{22}, and the four 256×11 arrays of indices Λ_{11}, Λ_{12}, Λ_{21}, Λ_{22}. At this point we start writing the coefficients and the phase indices in array form as we continue to compute them. When we finish, they will be in a format that is ready to be scanned or copied for numerical computations. For example, instead of writing $c_5c_4c_3s_2(-s_1)$ we will write $c_5{*}c_4{*}c_3{*}s_2{*}(-s_1)$.

Arrays of coefficients and phase indices for $P_{13,11}$. (32 terms each $P_{13,ij}$)

$P_{13,11} = [13]\{\, c_5P_{11,11} + s_5P_{11,21} \,\}$

$\tfrac{1}{2} * [\ t_{11} * [\ c_5{*}c_4{*}c_3{*}c_2{*}c_1$ [1,3,5,7,9,15,11,13]

$c_5{*}c_4{*}c_3{*}s_2{*}(-s_1)$ [1,4,5,7,9,15,11,13]

$c_5{*}c_4{*}s_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,7,9,15,11,13]
$c_5{*}c_4{*}s_3{*}\varepsilon_2{*}c_2{*}(-s_1)$	[1,4,6,7,9,15,11,13]
$c_5{*}s_4{*}(-s_3){*}c_2{*}c_1$	[1,3,5,8,9,15,11,13]
$c_5{*}s_4{*}(-s_3){*}s_2{*}(-s_1)$	[1,4,5,8,9,15,11,13]
$c_5{*}s_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,8,9,15,11,13]
$c_5{*}s_4{*}c_3{*}\varepsilon_2{*}c_2{*}(-s_1)$]	[1,4,6,8,9,15,11,13]
t_{12} * [$c_5{*}(-s_4){*}c_3{*}c_2{*}c_1$	[1,3,5,7,10,16,11,13]
$c_5{*}(-s_4){*}c_3{*}s_2{*}(-s_1)$	[1,4,5,7,10,16,11,13]
$c_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,7,10,16,11,13]
$c_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}c_2{*}(-s_1)$	[1,4,6,7,10,16,11,13]
$c_5{*}c_4{*}(-s_3){*}c_2{*}c_1$	[1,3,5,8,10,16,11,13]
$c_5{*}c_4{*}(-s_3){*}s_2{*}(-s_1)$	[1,4,5,8,10,16,11,13]
$c_5{*}c_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,8,10,16,11,13]
$c_5{*}c_4{*}c_3{*}\varepsilon_2{*}c_2{*}(-s_1)$]	[1,4,6,8,10,16,11,13]
t_{21} * [$s_5{*}c_4{*}c_3{*}c_2{*}c_1$	[1,3,5,7,9,17,12,13]
$s_5{*}c_4{*}c_3{*}s_2{*}(-s_1)$	[1,4,5,7,9,17,12,13]
$s_5{*}c_4{*}s_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,7,9,17,12,13]
$s_5{*}c_4{*}s_3{*}\varepsilon_2{*}c_2{*}(-s_1)$	[1,4,6,7,9,17,12,13]
$s_5{*}s_4{*}(-s_3){*}c_2{*}c_1$	[1,3,5,8,9,17,12,13]
$s_5{*}s_4{*}(-s_3){*}s_2{*}(-s_1)$	[1,4,5,8,9,17,12,13]
$s_5{*}s_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,8,9,17,12,13]
$s_5{*}s_4{*}c_3{*}\varepsilon_2{*}c_2{*}(-s_1)$]	[1,4,6,8,9,17,12,13]
t_{22} * [$s_5{*}(-s_4){*}c_3{*}c_2{*}c_1$	[1,3,5,7,10,18,12,13]
$s_5{*}(-s_4){*}c_3{*}s_2{*}(-s_1)$	[1,4,5,7,10,18,12,13]
$s_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,7,10,18,12,13]
$s_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}c_2{*}(-s_1)$	[1,4,6,7,10,18,12,13]
$s_5{*}c_4{*}(-s_3){*}c_2{*}c_1$	[1,3,5,8,10,18,12,13]
$s_5{*}c_4{*}(-s_3){*}s_2{*}(-s_1)$	[1,4,5,8,10,18,12,13]
$s_5{*}c_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}c_1$	[1,3,6,8,10,18,12,13]
$s_5{*}c_4{*}c_3{*}\varepsilon_2{*}c_2{*}(-s_1)$]]	[1,4,6,8,10,18,12,13]

Arrays of coefficients and phase indices for $P_{13,12}$.

$P_{13,12} =$ [13]{ $c_5 P_{11,12} + s_5 P_{11,22}$ }

$\frac{1}{2}$ * [t_{11} * [$c_5{*}c_4{*}c_3{*}c_2{*}s_1{*}\varepsilon_1$	[2,3,5,7,9,15,11,13]
$c_5{*}c_4{*}c_3{*}s_2{*}c_1{*}\varepsilon_1$	[2,4,5,7,9,15,11,13]
$c_5{*}c_4{*}s_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$	[2,3,6,7,9,15,11,13]
$c_5{*}c_4{*}s_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1$	[2,4,6,7,9,15,11,13]

$c_5{*}s_4{*}(-s_3){*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,8,9,15,11,13]
$c_5{*}s_4{*}(-s_3){*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,8,9,15,11,13]
$c_5{*}s_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,8,9,15,11,13]
$c_5{*}s_4{*}c_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1$ [2,4,6,8,9,15,11,13]

$h{*}[\ c_5{*}(-s_4){*}c_3{*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,7,10,16,11,13]
$c_5{*}(-s_4){*}c_3{*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,7,10,16,11,13]
$c_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,7,10,16,11,13]
$c_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1$ [2,4,6,7,10,16,11,13]
$c_5{*}c_4{*}(-s_3){*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,8,10,16,11,13]
$c_5{*}c_4{*}(-s_3){*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,8,10,16,11,13]
$c_5{*}c_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,8,10,16,11,13]
$c_5{*}c_4{*}c_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1\ ]$ [2,4,6,8,10,16,11,13]

$h{*}[\ s_5{*}c_4{*}c_3{*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,7,9,17,12,13]
$s_5{*}c_4{*}c_3{*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,7,9,17,12,13]
$s_5{*}c_4{*}s_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,7,9,17,12,13]
$s_5{*}c_4{*}s_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1$ [2,4,6,7,9,17,12,13]
$s_5{*}s_4{*}(-s_3){*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,8,9,17,12,13]
$s_5{*}s_4{*}(-s_3){*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,8,9,17,12,13]
$s_5{*}s_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,8,9,17,12,13]
$s_5{*}s_4{*}c_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1\ ]$ [2,4,6,8,9,17,12,13]

$s_5{*}(-s_4){*}c_3{*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,7,10,18,12,13]
$s_5{*}(-s_4){*}c_3{*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,7,10,18,12,13]
$s_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,7,10,18,12,13]
$s_5{*}(-s_4){*}s_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1$ [2,4,6,7,10,18,12,13]
$s_5{*}c_4{*}(-s_3){*}c_2{*}s_1{*}\varepsilon_1$ [2,3,5,8,10,18,12,13]
$s_5{*}c_4{*}(-s_3){*}s_2{*}c_1{*}\varepsilon_1$ [2,4,5,8,10,18,12,13]
$s_5{*}c_4{*}c_3{*}\varepsilon_2{*}(-s_2){*}s_1{*}\varepsilon_1$ [2,3,6,8,10,18,12,13]
$s_5{*}c_4{*}c_3{*}\varepsilon_2{*}c_2{*}c_1{*}\varepsilon_1\ ]$ [2,4,6,8,10,18,12,13]

Arrays of coefficients and phase indices for $P_{13,21}$.

$P_{13,21} =$ [14]{ $-s_5 P_{11,11} + c_5 P_{11,21}$ }

$\frac{1}{2}{*}[\ t_{11}{*}[\ (-s_5){*}c_4{*}c_3{*}c_2{*}c_1$ [1,3,5,7,9,15,11,14]
$(-s_5){*}c_4{*}c_3{*}s_2{*}(-s_1)$ [1,4,5,7,9,15,11,14]
$(-s_5){*}c_4{*}s_3{*}\varepsilon_2{*}(-s_2){*}c_1$ [1,3,6,7,9,15,11,14]
$(-s_5){*}c_4{*}s_3{*}\varepsilon_2{*}c_2{*}(-s_1)$ [1,4,6,7,9,15,11,14]
$(-s_5){*}s_4{*}(-s_3){*}c_2{*}c_1$ [1,3,5,8,9,15,11,14]
$(-s_5){*}s_4{*}(-s_3){*}s_2{*}(-s_1)$ [1,4,5,8,9,15,11,14]

$(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,14]
$(-s_5)*s_4*c_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,8,9,15,11,14]
$h*[\ (-s_5)*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,14]
$(-s_5)*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,14]
$(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,14]
$(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,14]
$(-s_5)*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,14]
$(-s_5)*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,14]
$(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,14]
$(-s_5)*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,16,11,14]
$h*[\ c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14]
$c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14]
$c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14]
$c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14]
$c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14]
$c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14]
$c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14]
$c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,14]
$c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14]
$c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14]
$c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14]
$c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14]
$c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14]
$c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14]
$c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14]
$c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,18,12,14]

Arrays of coefficients and phase indices for $P_{13,22}$.

$P_{13,22} =$ [14]$\{ -s_5P_{11,12} + c_5P_{11,22} \}$

$\frac{1}{2} * [\ t_{11} * [\ (-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14]
$(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14]
$(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14]
$(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14]
$(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14]
$(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14]
$(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14]
$(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,8,9,15,11,14]

$h*[\ (-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14]
$(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14]
$(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14]
$(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14]
$(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14]
$(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14]
$(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14]
$(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14]

$h*[\ c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14]
$c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14]
$c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14]
$c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14]
$c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14]
$c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14]
$c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14]
$c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,14]

$c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14]
$c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14]
$c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14]
$c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14]
$c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14]
$c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14]
$c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14]
$c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,14]

$$P_{15} = M_2 R_6 P_{13} = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{i\phi_{19}} & 0 \\ 0 & \varepsilon_2 e^{i\phi_{20}} \end{bmatrix}\begin{bmatrix} c_6 & s_6 \\ -s_6 & c_6 \end{bmatrix} P_{13} = \frac{1}{\sqrt{2}}\begin{bmatrix} c_6[19] & s_6[19] \\ \varepsilon_2(-s_6)[20] & \varepsilon_2 c_6[20] \end{bmatrix} P_{13}$$

Thus,

$$P_{15,11} = \tfrac{1}{\sqrt{2}}[19]\{\ c_6 P_{13,11} + s_6 P_{13,21}\ \}$$
$$P_{15,12} = \tfrac{1}{\sqrt{2}}[19]\{\ c_6 P_{13,12} + s_6 P_{13,22}\ \}$$
$$P_{15,21} = \tfrac{1}{\sqrt{2}}[20]\varepsilon_2\{\ (-s_6) P_{13,11} + c_6 P_{13,21}\ \}$$
$$P_{15,22} = \tfrac{1}{\sqrt{2}}[20]\varepsilon_2\{\ (-s_6) P_{13,12} + c_6 P_{13,22}\ \}$$

Arrays of coefficients and phase indices for $P_{15,11}$. (64 terms each $P_{15,\mathrm{ij}}$)

$P_{15,11} = [19]\{\, c_6 P_{13,11} + s_6 P_{13,21} \,\}$

$\frac{1}{2\sqrt{2}} * [\, t_{11} * [\, c_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,19]
$c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,19]
$c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,19]
$c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,19]
$c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,19]
$c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,19]
$c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,19]
$c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \;]$	[1,4,6,8,9,15,11,13,19]
$t_{12} * [\; c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,19]
$c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,19]
$c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,19]
$c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,19]
$c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,19]
$c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,19]
$c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,19]
$c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \;]$	[1,4,6,8,10,16,11,13,19]
$h * [\, c_6 * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,19]
$c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,19]
$c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,19]
$c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,19]
$c_6 * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,19]
$c_6 * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,19]
$c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,19]
$c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \;]$	[1,4,6,8,9,17,12,13,19]
$c_6 * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,19]
$c_6 * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,19]
$c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,19]
$c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,19]
$c_6 * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,19]
$c_6 * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,19]
$c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,19]
$c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \;]\;]$	[1,4,6,8,10,18,12,13,19]
$\frac{1}{2\sqrt{2}} * [\, t_{11} * [\, s_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,19]

	$s_6*(-s_5)*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,14,19]
	$s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,14,19]
	$s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,14,19]
	$s_6*(-s_5)*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,14,19]
	$s_6*(-s_5)*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,14,19]
	$s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,14,19]
	$s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,9,15,11,14,19]
t_{12} * [	$s_6*(-s_5)*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,14,19]
	$s_6*(-s_5)*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,14,19]
	$s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,14,19]
	$s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,14,19]
	$s_6*(-s_5)*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,14,19]
	$s_6*(-s_5)*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,14,19]
	$s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,14,19]
	$s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,10,16,11,14,19]
t_{21} * [	$s_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14,19]
	$s_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14,19]
	$s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14,19]
	$s_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14,19]
	$s_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14,19]
	$s_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14,19]
	$s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14,19]
	$s_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,9,17,12,14,19]
	$s_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14,19]
	$s_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14,19]
	$s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14,19]
	$s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14,19]
	$s_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14,19]
	$s_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14,19]
	$s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14,19]
	$s_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)$]]	[1,4,6,8,10,18,12,14,19]

Arrays of coefficients and phase indices for $P_{15,12}$.

$$P_{15,12} = [19]\{\, c_6 P_{13,12} + s_6 P_{13,22} \,\}$$

$\frac{1}{2\sqrt{2}}$ *[t_{11}*[	$c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,19]
	$c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,19]
	$c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,19]

$c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,15,11,13,19]
$c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,15,11,13,19]
$c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,15,11,13,19]
$c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,15,11,13,19]
$c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$ [2,4,6,8,9,15,11,13,19]

$t_{12} * [\; c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,16,11,13,19]
$c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,16,11,13,19]
$c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,16,11,13,19]
$c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,16,11,13,19]
$c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,16,11,13,19]
$c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,16,11,13,19]
$c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,10,16,11,13,19]
$c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$ [2,4,6,8,10,16,11,13,19]

$t_{21} * [\; c_6 * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,9,17,12,13,19]
$c_6 * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,9,17,12,13,19]
$c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,17,12,13,19]
$c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,17,12,13,19]
$c_6 * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,17,12,13,19]
$c_6 * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,17,12,13,19]
$c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,17,12,13,19]
$c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$ [2,4,6,8,9,17,12,13,19]

$t_{22} * [\; c_6 * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,18,12,13,19]
$c_6 * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,18,12,13,19]
$c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,18,12,13,19]
$c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,18,12,13,19]
$c_6 * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,18,12,13,19]
$c_6 * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,18,12,13,19]
$c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,10,18,12,13,19]
$c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]\;]$ [2,4,6,8,10,18,12,13,19]

$\frac{1}{2\sqrt{2}} * [\; t_{11} * [\; s_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,9,15,11,14,19]
$s_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,9,15,11,14,19]
$s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,15,11,14,19]
$s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,15,11,14,19]
$s_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,15,11,14,19]
$s_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,15,11,14,19]
$s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,15,11,14,19]
$s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$ [2,4,6,8,9,15,11,14,19]

$t_{12} * [$ $s_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,10,16,11,14,19]
$s_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,10,16,11,14,19]
$s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,10,16,11,14,19]
$s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,10,16,11,14,19]
$s_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,10,16,11,14,19]
$s_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,10,16,11,14,19]
$s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,10,16,11,14,19]
$s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$] [2,4,6,8,10,16,11,14,19]

$t_{21} * [$ $s_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,9,17,12,14,19]
$s_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,9,17,12,14,19]
$s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,9,17,12,14,19]
$s_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,9,17,12,14,19]
$s_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,9,17,12,14,19]
$s_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,9,17,12,14,19]
$s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,9,17,12,14,19]
$s_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$] [2,4,6,8,9,17,12,14,19]

$t_{22} * [$ $s_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,10,18,12,14,19]
$s_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,10,18,12,14,19]
$s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,10,18,12,14,19]
$s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,10,18,12,14,19]
$s_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,10,18,12,14,19]
$s_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,10,18,12,14,19]
$s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,10,18,12,14,19]
$s_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]] [2,4,6,8,10,18,12,14,19]

Arrays of coefficients and phase indices for $P_{15,21}$.

$P_{15,21} =$ [20]$\varepsilon_2\{ (-s_6)P_{13,11} + c_6 P_{13,21} \}$

$\frac{1}{2\sqrt{2}} *[$ $t_{11}*[$ $\varepsilon_2*(-s_6)*c_5*c_4*c_3*c_2*c_1$ [1,3,5,7,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*c_4*c_3*s_2*(-s_1)$ [1,4,5,7,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$ [1,3,6,7,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$ [1,4,6,7,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*c_2*c_1$ [1,3,5,8,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*s_2*(-s_1)$ [1,4,5,8,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$ [1,3,6,8,9,15,11,13,20]
$\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)$] [1,4,6,8,9,15,11,13,20]

$t_{12} * [$	$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,13,20]
$t_{21} * [$	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,13,20]
$t_{22} * [$	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]\ ]$	[1,4,6,8,10,18,12,13,20]
$\frac{1}{2\sqrt{2}} * [\ t_{11} * [$	$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,14,20]
$t_{12} * [$	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,14,20]

$\varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,16,11,14,20]
$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,16,11,14,20]
$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,10,16,11,14,20]

$t_{21} * [\ \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,9,17,12,14,20]

$t_{22} * [\ \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]\ ]$ [1,4,6,8,10,18,12,14,20]

Arrays of coefficients and phase indices for $P_{15,22}$.

$P_{15,22} = [20]\varepsilon_2 \{ (-s_6) P_{13,12} + c_6 P_{13,22} \}$

$\frac{1}{2\sqrt{2}} * [\ t_{11} * [\ \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,15,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1\ ]$ [2,4,6,8,9,15,11,13,20]

$t_{12} * [\ \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,16,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,16,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,16,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,16,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,16,11,13,20]
$\varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,16,11,13,20]

	$\varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,13,20]
	$\varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,13,20]
$t_{21} * [$	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,13,20]
$t_{22} * [$	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,13,20]
	$\varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;] \;]$	[2,4,6,8,10,18,12,13,20]
$\frac{1}{2\sqrt{2}} * [\; t_{11} * [$	$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,14,20]
$t_{12} * [$	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,14,20]
	$\varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,14,20]
$t_{21} * [$	$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,14,20]
	$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,14,20]

$\varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,17,12,14,20]
$\varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$ [2,4,6,8,9,17,12,14,20]

$t_{22} * [\;$ $\varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,10,18,12,14,20]
$\varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]\;]$ [2,4,6,8,10,18,12,14,20]

$$P_{17} = U_1 R_2^T P_{15} = \begin{bmatrix} e^{i\phi_3} & 0 \\ 0 & e^{i\phi_4} \end{bmatrix} \begin{bmatrix} c_2 & -s_2 \\ s_2 & c_2 \end{bmatrix} P_{15} = \begin{bmatrix} c_2[3] & -s_2[3] \\ s_2[4] & c_2[4] \end{bmatrix} P_{15}$$

Thus,

$P_{17,11} = c_2\,[3]\, P_{15,11} + (-s_2)\,[3]\, P_{15,21}$
$P_{17,12} = c_2\,[3]\, P_{15,12} + (-s_2)\,[3]\, P_{15,22}$
$P_{17,21} = s_2\,[4]\, P_{15,11} + c_2\,[4]\, P_{15,21}$
$P_{17,22} = s_2\,[4]\, P_{15,12} + c_2\,[4]\, P_{15,22}$

Arrays of coefficients and phase indices for $P_{17,11}$. (128 terms each $P_{17,ij}$)

$P_{17,11} = c_2\,[3]\, P_{15,11} + (-s_2)\,[3]\, P_{15,21}$

$\frac{1}{2\sqrt{2}} * [\; t_{11} * [\;$ $c_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,15,11,13,19,3]
$c_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \;]$ [1,4,6,8,9,15,11,13,19,3]

$t_{12} * [$	$c_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,19,3]
	$c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,13,19,3]
$t_{21} * [$	$c_2 * c_6 * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,19,3]
	$c_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,13,19,3]
$t_{22} * [$	$c_2 * c_6 * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,19,3]
	$c_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,13,19,3]
$t_{11} * [$	$c_2 * s_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,19,3]
	$c_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,14,19,3]
$t_{12} * [$	$c_2 * s_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,14,19,3]
	$c_2 * s_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,14,19,3]
	$c_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,14,19,3]
	$c_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,14,19,3]
	$c_2 * s_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,14,19,3]
	$c_2 * s_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,14,19,3]

	$c_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,14,19,3]
	$c_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,16,11,14,19,3]
t_{21} * [	$c_2 * s_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,14,19,3]
	$c_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,17,12,14,19,3]
t_{22} * [	$c_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,18,12,14,19,3]
t_{11} * [	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,15,11,13,20,3]
t_{12} * [	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,16,11,13,20,3]
t_{21} * [	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,20,3]

	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,17,12,13,20,3]
t_{22} * [	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,18,12,13,20,3]
t_{11} * [	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,15,11,14,20,3]
t_{12} * [	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,16,11,14,20,3]
t_{21} * [	$(-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,14,20,3]
	$(-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,17,12,14,20,3]

$t_{22}*[\ (-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14,20,3]
$(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]\ ]$	[1,4,6,8,10,18,12,14,20,3]

Arrays of coefficients and phase indices for $P_{17,12}$.

$P_{17,12} = c_2\,[3]\,P_{15,12} + (-s_2)\,[3]\,P_{15,22}$

$\frac{1}{2\sqrt{2}}*[\ t_{11}*[\ c_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,19,3]
$c_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,19,3]
$c_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,19,3]
$c_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,19,3]
$c_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,19,3]
$c_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,19,3]
$c_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,19,3]
$c_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,13,19,3]
$t_{12}*[\ c_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,19,3]
$c_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,13,19,3]
$c_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,13,19,3]
$c_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,13,19,3]
$c_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,13,19,3]
$c_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,13,19,3]
$c_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,13,19,3]
$c_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,13,19,3]
$t_{21}*[\ c_2*c_6*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,13,19,3]
$c_2*c_6*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,13,19,3]
$c_2*c_6*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,13,19,3]
$c_2*c_6*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,13,19,3]
$c_2*c_6*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,13,19,3]
$c_2*c_6*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,13,19,3]
$c_2*c_6*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,13,19,3]
$c_2*c_6*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,13,19,3]

t_{22} * [	$c_2*c_6*s_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,13,19,3]
	$c_2*c_6*s_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,13,19,3]
	$c_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,13,19,3]
	$c_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,13,19,3]
	$c_2*c_6*s_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,13,19,3]
	$c_2*c_6*s_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,13,19,3]
	$c_2*c_6*s_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,13,19,3]
	$c_2*c_6*s_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,10,18,12,13,19,3]
t_{11} * [	$c_2*s_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14,19,3]
	$c_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,9,15,11,14,19,3]
t_{12} * [	$c_2*s_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,19,3]
	$c_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,10,16,11,14,19,3]
t_{21} * [	$c_2*s_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,19,3]
	$c_2*s_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,19,3]
	$c_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,19,3]
	$c_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,19,3]
	$c_2*s_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,19,3]
	$c_2*s_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,19,3]
	$c_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,19,3]
	$c_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,9,17,12,14,19,3]
t_{22} * [	$c_2*s_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,19,3]
	$c_2*s_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,19,3]
	$c_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,19,3]
	$c_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,19,3]

	$c_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,14,19,3]
	$c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$]	[2,4,6,8,10,18,12,14,19,3]
t_{11} * [	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$]	[2,4,6,8,9,15,11,13,20,3]
t_{12} * [	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$]	[2,4,6,8,10,16,11,13,20,3]
t_{21} * [	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$]	[2,4,6,8,9,17,12,13,20,3]
t_{22} * [	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,13,20,3]
	$(-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$]	[2,4,6,8,10,18,12,13,20,3]

$t_{11} * [$	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,14,20,3]
$t_{12} * [$	$(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14,20,3]
$t_{21} * [$	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,14,20,3]
$t_{22} * [$	$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14,20,3]
	$(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]\ ]$	[2,4,6,8,10,18,12,14,20,3]

Arrays of coefficients and phase indices for $P_{17,21}$.

$P_{17,21} = s_2\,[4]\,P_{15,11} + c_2\,[4]\,P_{15,21}$

$\frac{1}{2\sqrt{2}} * [\; t_{11} * [$	$s_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,19,4]
	$s_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\;]$	[1,4,6,8,9,15,11,13,19,4]
$t_{12} * [$	$s_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,19,4]
	$s_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\;]$	[1,4,6,8,10,16,11,13,19,4]
$t_{21} * [$	$s_2 * c_6 * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,19,4]
	$s_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\;]$	[1,4,6,8,9,17,12,13,19,4]
$t_{22} * [$	$s_2 * c_6 * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,19,4]
	$s_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\;]$	[1,4,6,8,10,18,12,13,19,4]
$t_{11} * [$	$s_2 * s_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,19,4]
	$s_2 * s_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,19,4]
	$s_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,19,4]
	$s_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,19,4]
	$s_2 * s_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,19,4]
	$s_2 * s_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,19,4]

	$s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,9,15,11,14,19,4]
t_{12} * [	$s_2*s_6*(-s_5)*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,10,16,11,14,19,4]
t_{21} * [	$s_2*s_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,9,17,12,14,19,4]
t_{22} * [	$s_2*s_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,10,18,12,14,19,4]
t_{11} * [	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,9,15,11,13,20,4]
t_{12} * [	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,13,20,4]

	$c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,16,11,13,20,4]
t_{21} * [	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,17,12,13,20,4]
t_{22} * [	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,18,12,13,20,4]
t_{11} * [	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,15,11,14,20,4]
t_{12} * [	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,16,11,14,20,4]

$t_{21} * [$	$c_2*\varepsilon_2*c_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,14,20,4]
$t_{22} * [$	$c_2*\varepsilon_2*c_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14,20,4]
	$c_2*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]\ ]$	[1,4,6,8,10,18,12,14,20,4]

Arrays of coefficients and phase indices for $P_{17,22}$.

$P_{17,22} = s_2\,[4]\,P_{15,12} + c_2\,[4]\,P_{15,22}$

$\frac{1}{2\sqrt{2}} *[\ t_{11}*[$	$s_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,19,4]
	$s_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,19,4]
	$s_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,19,4]
	$s_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,19,4]
	$s_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,19,4]
	$s_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,19,4]
	$s_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,19,4]
	$s_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,13,19,4]
$t_{12} * [$	$s_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,19,4]
	$s_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,13,19,4]
	$s_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,13,19,4]
	$s_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,13,19,4]
	$s_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,13,19,4]
	$s_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,13,19,4]
	$s_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,13,19,4]
	$s_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,13,19,4]

$t_{21} * [$	$s_2*c_6*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,13,19,4]
	$s_2*c_6*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,13,19,4]
	$s_2*c_6*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,13,19,4]
	$s_2*c_6*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,13,19,4]
	$s_2*c_6*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,13,19,4]
	$s_2*c_6*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,13,19,4]
	$s_2*c_6*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,13,19,4]
	$s_2*c_6*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,13,19,4]
$t_{22} * [$	$s_2*c_6*s_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,13,19,4]
	$s_2*c_6*s_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,13,19,4]
	$s_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,13,19,4]
	$s_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,13,19,4]
	$s_2*c_6*s_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,13,19,4]
	$s_2*c_6*s_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,13,19,4]
	$s_2*c_6*s_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,13,19,4]
	$s_2*c_6*s_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,13,19,4]
$t_{11} * [$	$s_2*s_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14,19,4]
	$s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,14,19,4]
$t_{12} * [$	$s_2*s_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,19,4]
	$s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14,19,4]
$t_{21} * [$	$s_2*s_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,19,4]
	$s_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,19,4]

	$s_2*s_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,19,4]
	$s_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,9,17,12,14,19,4]
t_{22} * [	$s_2*s_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14,19,4]
	$s_2*s_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,10,18,12,14,19,4]
t_{11} * [	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,9,15,11,13,20,4]
t_{12} * [	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,10,16,11,13,20,4]
t_{21} * [	$c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,13,20,4]
	$c_2*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,9,17,12,13,20,4]

$t_{22} * [$	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,13,20,4]
	$c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,18,12,13,20,4]
$t_{11} * [$	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,14,20,4]
$t_{12} * [$	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,14,20,4]
$t_{21} * [$	$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,14,20,4]
$t_{22} * [$	$c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,14,20,4]
	$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,14,20,4]

$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,18,12,14,20,4]
$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,10,18,12,14,20,4]
$c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$]] [2,4,6,8,10,18,12,14,20,4]

$$T = P_{19} = M_1 R_1^T P_{17} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi_{21}} & 0 \\ 0 & \varepsilon_1 e^{i\phi_{22}} \end{bmatrix} \begin{bmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{bmatrix} P_{17} = \frac{1}{\sqrt{2}} \begin{bmatrix} c_1[21] & -s_1[21] \\ \varepsilon_1 s_1[22] & \varepsilon_1 c_1[22] \end{bmatrix} P_{17}$$

Thus,

$T_{11} = \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,11} + (-s_1) [21] P_{17,21}\}$

$T_{12} = \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,12} + (-s_1) [21] P_{17,22}\}$

$T_{21} = \frac{1}{\sqrt{2}} \{\varepsilon_1 s_1 [22] P_{17,11} + \varepsilon_1 c_1 [22] P_{17,21}\}$

$T_{22} = \frac{1}{\sqrt{2}} \{\varepsilon_1 s_1 [22] P_{17,12} + \varepsilon_1 c_1 [22] P_{17,22}\}$

Arrays of coefficients and phase indices for T_{11}. (256 terms each T_{ij})

$T_{11} = \frac{1}{\sqrt{2}} \{c_1 [21] P_{17,11} + (-s_1) [21] P_{17,21}\}$

$\frac{1}{4} *$ [$t_{11} *$ [$c_1 * c_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,15,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$] [1,4,6,8,9,15,11,13,19,3,21]

$t_{12} *$ [$c_1 * c_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,16,11,13,19,3,21]
$c_1 * c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$] [1,4,6,8,10,16,11,13,19,3,21]

$t_{21} *$ [$c_1 * c_2 * c_6 * s_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,17,12,13,19,3,21]
$c_1 * c_2 * c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,17,12,13,19,3,21]

$c_1*c_2*c_6*s_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,13,19,3,21]
$t_{22} * [\ c_1*c_2*c_6*s_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,18,12,13,19,3,21]
$t_{11} * [\ c_1*c_2*s_6*(-s_5)*c_4*c_3*c_2*c_1$	[1,3,5,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,15,11,14,19,3,21]
$t_{12} * [\ c_1*c_2*s_6*(-s_5)*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,16,11,14,19,3,21]
$t_{21} * [\ c_1*c_2*s_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,14,19,3,21]

$t_{22} * [\ c_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,14,19,3,21]
$c_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,14,19,3,21]

$t_{11} * [\ c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,13,20,3,21]

$t_{12} * [\ c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,13,20,3,21]

$t_{21} * [\ c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,13,20,3,21]

$t_{22} * [\ c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,20,3,21]

$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,18,12,13,20,3,21]
$t_{11} *$ [$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,15,11,14,20,3,21]
$t_{12} *$ [$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,16,11,14,20,3,21]
$t_{21} *$ [$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,14,20,3,21
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,9,17,12,14,20,3,21]
$t_{22} *$ [$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,14,20,3,21]
$c_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)$]	[1,4,6,8,10,18,12,14,20,3,21]
$t_{11} *$ [$(-s_1) * s_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,19,4,21]
$(-s_1) * s_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,19,4,21]
$(-s_1) * s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,19,4,21]

$(-s_1)*s_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,15,11,13,19,4,21]
$t_{12} * [\,(-s_1)*s_2*c_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,16,11,13,19,4,21]
$t_{21} * [\,(-s_1)*s_2*c_6*s_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,13,19,4,21]
$t_{22} * [\,(-s_1)*s_2*c_6*s_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,18,12,13,19,4,21]
$t_{11} * [\,(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*c_2*c_1$	[1,3,5,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,15,11,14,19,4,21]
$t_{12} * [\,(-s_1)*s_2*s_6*(-s_5)*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,14,19,4,21]

$(-s_1)*s_2*s_6*(-s_5)*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,16,11,14,19,4,21]
$t_{21} * [\ (-s_1)*s_2*s_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,14,19,4,21]
$t_{22} * [\ (-s_1)*s_2*s_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,18,12,14,19,4,21]
$t_{11} * [\ (-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,15,11,13,20,4,21]
$t_{12} * [\ (-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,13,20,4,21]

$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,16,11,13,20,4,21]
$t_{21} * [\,(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,17,12,13,20,4,21]
$t_{22} * [\,(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,18,12,13,20,4,21]
$t_{11} * [\,(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*c_2*c_1$	[1,3,5,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,15,11,14,20,4,21]
$t_{12} * [\,(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,10,16,11,14,20,4,21]
$t_{21} * [\,(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,17,12,14,20,4,21]

$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)$]	[1,4,6,8,9,17,12,14,20,4,21]
t_{22} * [$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*(-s_1)$]]	[1,4,6,8,10,18,12,14,20,4,21]

Arrays of coefficients and phase indices for T_{12}.

$$T_{12} = \frac{1}{\sqrt{2}} \{c_1\,[21]\,P_{17,12} + (-s_1)\,[21]\,P_{17,22}\}$$

$\frac{1}{4}$ * [t_{11} * [$c_1*c_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,19,3,21]
$c_1*c_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,9,15,11,13,19,3,21]
t_{12} * [$c_1*c_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,13,19,3,21]
$c_1*c_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$]	[2,4,6,8,10,16,11,13,19,3,21]
t_{21} * [$c_1*c_2*c_6*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,13,19,3,21]

$c_1*c_2*c_6*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,13,19,3,21]
$c_1*c_2*c_6*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,13,19,3,21]
$t_{22} * [\ c_1*c_2*c_6*s_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,13,19,3,21]
$c_1*c_2*c_6*s_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,13,19,3,21]
$t_{11} * [\ c_1*c_2*s_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,14,19,3,21]
$t_{12} * [\ c_1*c_2*s_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,19,3,21]
$c_1*c_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14,19,3,21]
$t_{21} * [\ c_1*c_2*s_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,19,3,21]
$c_1*c_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,14,19,3,21]

$t_{22} * [\, c_1*c_2*s_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14,19,3,21]
$c_1*c_2*s_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1 \,]$	[2,4,6,8,10,18,12,14,19,3,21]
$t_{11} * [\, c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1 \,]$	[2,4,6,8,9,15,11,13,20,3,21]
$t_{12} * [\, c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,13,20,3,21]
$(c_1*-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1 \,]$	[2,4,6,8,10,16,11,13,20,3,21]
$t_{21} * [\, c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1 \,]$	[2,4,6,8,9,17,12,13,20,3,21]
$t_{22} * [\, c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,13,20,3,21]

$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,13,20,3,21]
$c_1*(-s_2)*\varepsilon_2*(-s_6)*s_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,13,20,3,21]
$t_{11}*[\ c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,14,20,3,21]
$t_{12}*[\ c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14,20,3,21]
$t_{21}*[\ c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,14,20,3,21]
$t_{22}*[\ c_1*(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14,20,3,21]
$c_1*(-s_2)*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,14,20,3,21]

$t_{11} * [\ (-s_1)*s_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,13,19,4,21]
$t_{12} * [\ (-s_1)*s_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,13,19,4,21]
$(-s_1)*s_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,13,19,4,21]
$t_{21} * [\ (-s_1)*s_2*c_6*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,13,19,4,21]
$t_{22} * [\ (-s_1)*s_2*c_6*s_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,13,19,4,21]
$(-s_1)*s_2*c_6*s_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,13,19,4,21]
$t_{11} * [\ (-s_1)*s_2*s_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,14,19,4,21]

$(-s_1)*s_2*s_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,14,19,4,21]
$t_{12}*[\,(-s_1)*s_2*s_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,19,4,21]
$(-s_1)*s_2*s_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14,19,4,21]
$t_{21}*[\,(-s_1)*s_2*s_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,14,19,4,21]
$t_{22}*[\,(-s_1)*s_2*s_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14,19,4,21]
$(-s_1)*s_2*s_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,18,12,14,19,4,21]
$t_{11}*[\,(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,15,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,15,11,13,20,4,21]
$t_{12}*[\,(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,16,11,13,20,4,21]

$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,10,16,11,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$] [2,4,6,8,10,16,11,13,20,4,21]

t_{21} * [$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,9,17,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$] [2,4,6,8,9,17,12,13,20,4,21]

t_{22} * [$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,10,18,12,13,20,4,21]
$(-s_1)*c_2*\varepsilon_2*(-s_6)*s_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$] [2,4,6,8,10,18,12,13,20,4,21]

t_{11} * [$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,8,9,15,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1$] [2,4,6,8,9,15,11,14,20,4,21]

t_{12} * [$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*c_2*s_1*\varepsilon_1$ [2,3,5,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*c_3*s_2*c_1*\varepsilon_1$ [2,4,5,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$ [2,3,6,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$ [2,4,6,7,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*c_2*s_1*\varepsilon_1$ [2,3,5,8,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*(-s_3)*s_2*c_1*\varepsilon_1$ [2,4,5,8,10,16,11,14,20,4,21]

$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,16,11,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*(-s_5)*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,10,16,11,14,20,4,21]
$t_{21} * [\ (-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,9,17,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]$	[2,4,6,8,9,17,12,14,20,4,21]
$t_{22} * [\ (-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*c_3*c_2*s_1*\varepsilon_1$	[2,3,5,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*c_3*s_2*c_1*\varepsilon_1$	[2,4,5,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*c_1*\varepsilon_1$	[2,4,6,7,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*(-s_3)*c_2*s_1*\varepsilon_1$	[2,3,5,8,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*(-s_3)*s_2*c_1*\varepsilon_1$	[2,4,5,8,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*(-s_2)*s_1*\varepsilon_1$	[2,3,6,8,10,18,12,14,20,4,21]
$(-s_1)*c_2*\varepsilon_2*c_6*c_5*c_4*c_3*\varepsilon_2*c_2*c_1*\varepsilon_1\ ]\ ]$	[2,4,6,8,10,18,12,14,20,4,21]

Arrays of coefficients and phase indices for T_{21}.

$$T_{21} = \frac{1}{\sqrt{2}} \{\varepsilon_1 s_1\ [22]\ P_{17,11} + \varepsilon_1 c_1\ [22]\ P_{17,21}\}$$

$\frac{1}{4} * [\ t_{11} * [\ \varepsilon_1*s_1*c_2*c_6*c_5*c_4*c_3*c_2*c_1$	[1,3,5,7,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*c_4*c_3*s_2*(-s_1)$	[1,4,5,7,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*c_4*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*c_4*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*s_4*(-s_3)*c_2*c_1$	[1,3,5,8,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*s_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*s_4*c_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,8,9,15,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*s_4*c_3*\varepsilon_2*c_2*(-s_1)\ ]$	[1,4,6,8,9,15,11,13,19,3,22]
$t_{12} * [\ \varepsilon_1*s_1*c_2*c_6*c_5*(-s_4)*c_3*c_2*c_1$	[1,3,5,7,10,16,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*(-s_4)*c_3*s_2*(-s_1)$	[1,4,5,7,10,16,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*(-s_2)*c_1$	[1,3,6,7,10,16,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*(-s_4)*s_3*\varepsilon_2*c_2*(-s_1)$	[1,4,6,7,10,16,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*c_4*(-s_3)*c_2*c_1$	[1,3,5,8,10,16,11,13,19,3,22]
$\varepsilon_1*s_1*c_2*c_6*c_5*c_4*(-s_3)*s_2*(-s_1)$	[1,4,5,8,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,10,16,11,13,19,3,22]

$t_{21} * [\ \varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,9,17,12,13,19,3,22]

$t_{22} * [\ \varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,10,18,12,13,19,3,22]

$t_{11} * [\ \varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,9,15,11,14,19,3,22]

$t_{12} * [\ \varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,10,16,11,14,19,3,22]

$t_{21} * [\ \varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,17,12,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,17,12,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,14,19,3,22]
$t_{22} * [\ \varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,14,19,3,22]
$t_{11} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,13,20,3,22]
$t_{12} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,13,20,3,22]
$t_{21} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,20,3,22]

$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,9,17,12,13,20,3,22]

$t_{22} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,10,18,12,13,20,3,22]

$t_{11} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,9,15,11,14,20,3,22]

$t_{12} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,10,16,11,14,20,3,22]

$t_{21} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$ [1,4,6,8,9,17,12,14,20,3,22]

$t_{22} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,18,12,14,20,3,22]

$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,14,20,3,22]
$t_{11} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,13,19,4,22]
$t_{12} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,13,19,4,22]
$t_{21} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,13,19,4,22]
$t_{22} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,13,19,4,22]

$t_{11} * [\ \varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,14,19,4,22]
$t_{12} * [\ \varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,14,19,4,22]
$t_{21} * [\ \varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,14,19,4,22]
$t_{22} * [\ \varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,14,19,4,22]
$t_{11} * [\ \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,13,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,13,20,4,22]
$t_{12} * [\ \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,16,11,13,20,4,22]
$t_{21} * [\ \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,17,12,13,20,4,22]
$t_{22} * [\ \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * c_1$	[1,3,5,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * (-s_1)$	[1,4,5,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,10,18,12,13,20,4,22]
$t_{11} * [\ \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * c_1$	[1,3,5,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * (-s_1)$	[1,4,5,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$	[1,4,6,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * c_1$	[1,3,5,8,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * (-s_1)$	[1,4,5,8,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$	[1,3,6,8,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1)\ ]$	[1,4,6,8,9,15,11,14,20,4,22]

$t_{12} * [\, \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,16,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \,]$ [1,4,6,8,10,16,11,14,20,4,22]

$t_{21} * [\, \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * c_1$ [1,3,5,7,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * (-s_1)$ [1,4,5,7,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,9,17,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \,]$ [1,4,6,8,9,17,12,14,20,4,22]

$t_{22} * [\, \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * c_1$ [1,3,5,7,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * (-s_1)$ [1,4,5,7,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,7,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * (-s_1)$ [1,4,6,7,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * c_1$ [1,3,5,8,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * (-s_1)$ [1,4,5,8,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * c_1$ [1,3,6,8,10,18,12,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * (-s_1) \,]\,]$ [1,4,6,8,10,18,12,14,20,4,22]

Arrays of coefficients and phase indices for T_{22}.

$$T_{22} = \frac{1}{\sqrt{2}} \{\varepsilon_1 s_1\, [22]\, P_{17,12} + \varepsilon_1 c_1\, [22]\, P_{17,22}\}$$

$\frac{1}{4} * [\, t_{11} * [\, \varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,15,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \,]$ [2,4,6,8,9,15,11,13,19,3,22]

$t_{12} * [\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,10,16,11,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1]$ [2,4,6,8,10,16,11,13,19,3,22]

$t_{21} * [\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,17,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1]$ [2,4,6,8,9,17,12,13,19,3,22]

$t_{22} * [\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,10,18,12,13,19,3,22]

$\varepsilon_1 * s_1 * c_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1]$ [2,4,6,8,10,18,12,13,19,3,22]

$t_{11} * [\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$ [2,4,5,8,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,8,9,15,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1]$ [2,4,6,8,9,15,11,14,19,3,22]

$t_{12} * [\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$ [2,3,5,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$ [2,4,5,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$ [2,3,6,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$ [2,4,6,7,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$ [2,3,5,8,10,16,11,14,19,3,22]

$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,14,19,3,22]
$t_{21} * [\, \varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,14,19,3,22]
$t_{22} * [\, \varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,14,19,3,22]
$\varepsilon_1 * s_1 * c_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,18,12,14,19,3,22]
$t_{11} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,13,20,3,22]
$t_{12} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,13,20,3,22]
$t_{21} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,13,20,3,22]

$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,13,20,3,22]
$t_{22} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,13,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,18,12,13,20,3,22]
$t_{11} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,14,20,3,22]
$t_{12} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,14,20,3,22]
$t_{21} * [\, \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,14,20,3,22]

$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1\ ]$	[2,4,6,8,9,17,12,14,20,3,22]
$t_{22} * [\ \varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,14,20,3,22]
$\varepsilon_1 * s_1 * (-s_2) * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1\ ]$	[2,4,6,8,10,18,12,14,20,3,22]
$t_{11} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1\ ]$	[2,4,6,8,9,15,11,13,19,4,22]
$t_{12} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1\ ]$	[2,4,6,8,10,16,11,13,19,4,22]
$t_{21} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1\ ]$	[2,4,6,8,9,17,12,13,19,4,22]
$t_{22} * [\ \varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,13,19,4,22]

$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,13,19,4,22]
$\varepsilon_1 * c_1 * s_2 * c_6 * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,18,12,13,19,4,22]
$t_{11} * [\; \varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,14,19,4,22]
$t_{12} * [\; \varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,14,19,4,22]
$t_{21} * [\; \varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,14,19,4,22]
$t_{22} * [\; \varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,14,19,4,22]

$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,14,19,4,22]
$\varepsilon_1 * c_1 * s_2 * s_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,18,12,14,19,4,22]
$t_{11} * [\; \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,13,20,4,22]
$t_{12} * [\; \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,13,20,4,22]
$t_{21} * [\; \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,13,20,4,22]
$t_{22} * [\; \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,13,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * (-s_6) * s_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,18,12,13,20,4,22]
$t_{11} * [\; \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,15,11,14,20,4,22]

$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,15,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,15,11,14,20,4,22]
$t_{12} * [\, \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,16,11,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * (-s_5) * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,10,16,11,14,20,4,22]
$t_{21} * [\, \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,9,17,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * s_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]$	[2,4,6,8,9,17,12,14,20,4,22]
$t_{22} * [\, \varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * c_2 * s_1 * \varepsilon_1$	[2,3,5,7,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * c_3 * s_2 * c_1 * \varepsilon_1$	[2,4,5,7,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,7,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * (-s_4) * s_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1$	[2,4,6,7,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * c_2 * s_1 * \varepsilon_1$	[2,3,5,8,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * (-s_3) * s_2 * c_1 * \varepsilon_1$	[2,4,5,8,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * (-s_2) * s_1 * \varepsilon_1$	[2,3,6,8,10,18,12,14,20,4,22]
$\varepsilon_1 * c_1 * c_2 * \varepsilon_2 * c_6 * c_5 * c_4 * c_3 * \varepsilon_2 * c_2 * c_1 * \varepsilon_1 \;]\;]$	[2,4,6,8,10,18,12,14,20,4,22]

Appendix B
MATLAB PROGRAMS.

Our Matlab programs compute the output intensity of the general gyroscope described in Section 5 for three types of frequency distribution functions *g*; a Gaussian function, a step function, and a tent function, using Equations 3.11, 3.12, and 3.15, respectively. In order to compute the intensity given by any one of these three equations we need the four arrays of coefficients C_{ij} $(i, j = 1,2)$ and the sets of phase indices $\Lambda_{ij}(s)$ to compute all the phase shifts $\beta_{ij}(s)$ for all $i, j = 1, 2$ and $1 \leq s \leq 256$.

The four arrays of coefficients C_{ij} $(i,j = 1,2)$ are provided by the four functions **Coef11_arr, Coef12_arr, Coef21_arr,** and **Coef22_arr**. We show part of the function **Coef11_arr** to illustrate it's structure. The complete lists of coefficients for T_{11}, T_{12}, T_{21}, and T_{22} can be found in Appendix 1.

```
function Coef11 = Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e1,e2,t)

% Array of coefficients for T11.

t11=t(1);
t12=t(2);
t21=t(3);
t22=t(4);

Coef11 = (1/4) * [

        t11 * [ c1*c2*c6*c5*c4*c3*c2*c1
                c1*c2*c6*c5*c4*c3*s2*(-s1)
                c1*c2*c6*c5*c4*s3*e2*(-s2)*c1
                c1*c2*c6*c5*c4*s3*e2*c2*(-s1)
                c1*c2*c6*c5*s4*(-s3)*c2*c1
                c1*c2*c6*c5*s4*(-s3)*s2*(-s1)
                c1*c2*c6*c5*s4*c3*e2*(-s2)*c1
                c1*c2*c6*c5*s4*c3*e2*c2*(-s1) ]
```

```
t12 * [ c1*c2*c6*c5*(-s4)*c3*c2*c1
        c1*c2*c6*c5*(-s4)*c3*s2*(-s1)
        c1*c2*c6*c5*(-s4)*s3*e2*(-s2)*c1
        c1*c2*c6*c5*(-s4)*s3*e2*c2*(-s1)
        .
        .
        .
t21 * [ (-s1)*c2*e2*c6*c5*c4*c3*c2*c1
        (-s1)*c2*e2*c6*c5*c4*c3*s2*(-s1)
        (-s1)*c2*e2*c6*c5*c4*s3*e2*(-s2)*c1
        (-s1)*c2*e2*c6*c5*c4*s3*e2*c2*(-s1)
        (-s1)*c2*e2*c6*c5*s4*(-s3)*c2*c1
        (-s1)*c2*e2*c6*c5*s4*(-s3)*s2*(-s1)
        (-s1)*c2*e2*c6*c5*s4*c3*e2*(-s2)*c1
        (-s1)*c2*e2*c6*c5*s4*c3*e2*c2*(-s1) ]

t22 * [ (-s1)*c2*e2*c6*c5*(-s4)*c3*c2*c1
        (-s1)*c2*e2*c6*c5*(-s4)*c3*s2*(-s1)
        (-s1)*c2*e2*c6*c5*(-s4)*s3*e2*(-s2)*c1
        (-s1)*c2*e2*c6*c5*(-s4)*s3*e2*c2*(-s1)
        (-s1)*c2*e2*c6*c5*c4*(-s3)*c2*c1
        (-s1)*c2*e2*c6*c5*c4*(-s3)*s2*(-s1)
        (-s1)*c2*e2*c6*c5*c4*c3*e2*(-s2)*c1
        (-s1)*c2*e2*c6*c5*c4*c3*e2*c2*(-s1) ] ];
```

The inputs to **Coef11_arr** are s1, *s2, s3, s4, s5, s6, c1, c2, c3, c4, c5, c6, e1, e2, t11, t12, t21,* and *t22,* where

$$
\begin{aligned}
s1 &= \sin(a(1)), & c1 &= \cos(a(1)),\\
s2 &= \sin(a(2)), & c2 &= \cos(a(2)),\\
s3 &= \sin(A1 + a(3)), & c3 &= \cos(A1 + a(3)),\\
s4 &= \sin(\tfrac{\pi}{4} + b(1)), & c4 &= \cos(\tfrac{\pi}{4} + b(1)),\\
s5 &= \sin(\tfrac{\pi}{4} + b(2)), & c5 &= \cos(\tfrac{\pi}{4} + b(2)),\\
s6 &= \sin(A2 + a(4)), & c6 &= \cos(A2 + a(4)),
\end{aligned}
$$

$A1 = 0^\circ$ or 90°, $A2 = 0^\circ$ or 90°, and *a(1), a(2), a(3), a(4), b(1), b(2)* are arbitrary angles. The two polarization extinction ratios are $e1 = \varepsilon_1$ and $e2 = \varepsilon_2$. The magnitudes of the entries of the coil's 2×2 complex matrix are *t11, t12, t21,* and *t22.*

The sets of phase indices $\Lambda_{ij}(s)$ $(1 \le s \le 256)$ for a particular index pair *(i, j)* have been arranged in an 11×256 array *Lij*. In our Matlab programs these four arrays are produced by four functions called **Ph_arrCo11**, **Ph_arrCo12**, **Ph_arrCo21**,

Ph_arrCo22. We show part of the function **Ph_arrCo11** to illustrate it's structure. The complete lists of phase indices for T_{11}, T_{12}, T_{21}, and T_{22} can be found in Appendix 1.

```
function L11=Ph_arrCo11

L11 = [      [1,3,5,7,9,15,11,13,19,3,21]
             [1,4,5,7,9,15,11,13,19,3,21]
             [1,3,6,7,9,15,11,13,19,3,21]
             [1,4,6,7,9,15,11,13,19,3,21]
             [1,3,5,8,9,15,11,13,19,3,21]
             [1,4,5,8,9,15,11,13,19,3,21]
             [1,3,6,8,9,15,11,13,19,3,21]
             [1,4,6,8,9,15,11,13,19,3,21]

             [1,3,5,7,10,16,11,13,19,3,21]
             [1,4,5,7,10,16,11,13,19,3,21]
             [1,3,6,7,10,16,11,13,19,3,21]
             [1,4,6,7,10,16,11,13,19,3,21]
             [1,3,5,8,10,16,11,13,19,3,21]
               .
               .
               .
             [1,3,5,7,9,17,12,14,20,4,21]
             [1,4,5,7,9,17,12,14,20,4,21]
             [1,3,6,7,9,17,12,14,20,4,21]
             [1,4,6,7,9,17,12,14,20,4,21]
             [1,3,5,8,9,17,12,14,20,4,21]
             [1,4,5,8,9,17,12,14,20,4,21]
             [1,3,6,8,9,17,12,14,20,4,21]
             [1,4,6,8,9,17,12,14,20,4,21]

             [1,3,5,7,10,18,12,14,20,4,21]
             [1,4,5,7,10,18,12,14,20,4,21]
             [1,3,6,7,10,18,12,14,20,4,21]
             [1,4,6,7,10,18,12,14,20,4,21]
             [1,3,5,8,10,18,12,14,20,4,21]
             [1,4,5,8,10,18,12,14,20,4,21]
             [1,3,6,8,10,18,12,14,20,4,21]
             [1,4,6,8,10,18,12,14,20,4,21] ];
```

For example, *L11(2, :)* = [1,4,5,7,9,15,11,13,19,3,21] and *L11(s,:)* contains the eleven indices needed to define the phase shifts

$$\beta_{11}(s)\omega_1 = \omega_1 \sum_{r\in L11(s,:)} \phi_r \quad (1 \le s \le 256). \tag{A1}$$

These summations are implemented in Matlab as follows. We define a 22-column vector called $phase = [\,\omega_1\phi_1 \;\; \omega_1\phi_2 \;\; \omega_1\phi_3 \;\; \ldots \;\; \omega_1\phi_{22}\,]$ that contains all the phase shifts involved in the clockwise path of the gyro: $\phi_r = 0$ for r odd, $\phi_r = (n_r - n_{r-1})l_{\frac{r}{2}}/c$ for r even, r = 1, 2, 3, ... 14, and r = 19, 20, 21, 22. The four taus, τ_{11}, τ_{12}, τ_{21}, and τ_{22} representing the coil phase shifts receive the indices 15, 16, 17, and 18, respectively; that is $\phi_{15} = \tau_{11}$, $\phi_{16} = \tau_{12}$, $\phi_{17} = \tau_{21}$, and $\phi_{18} = \tau_{22}$. The values of all 256 $\beta_{ij}(s)$ $(1 \le s \le 256)$ are contained in a 256-row vector called *Bij* (*i*, *j* = 1, 2). In order to speed up the program we use the vector/matrix operations available in Matlab. To evaluate the sums in Equation A1 for all *s* simultaneously we form the matrix

*Phase_M = phase*ones(1, 256)*

which is a 22×256 matrix whose columns are all equal to the 22-column vector *phase*. Then we call the function **Ph_arrCoij** and let

Lij = **Ph_arrCoij** '

so that *Lij* is an 11×256 array whose columns contain the indices $\Lambda_{ij}(s)$ $(1 \le s \le 256)$. The statement

Phase_M(Lij)

produces an 11×256 array whose *s*-th column contains the phases ϕ_r for $r \in \Lambda_{ij}(s)$, $1 \le s \le 256$. Next, the statement

Bij = sum(*Phase_M(Lij)*)

sums the elements of the columns of *Phase_M(Lij)* to produce the 256-row vector *Bij* containing the values of $\beta_{ij}(s)$ defined by Equation A1.

Next, we describe the function **StepIntens** where the above statements appear. This function computes the output intensity of the gyroscope for several values of the lengths

l_4 and l_7 when g is a step function. The inputs to **StepIntens** are *Del, l, e, a, b, t, tau, phi, input, w1, N,* and *M.*

The differences (deltas) in the index of refraction of the two polarization modes of the PM fibers and IOCs are contained in the 7-vector *Del* ; $Del(i) = n_{2i} - n_{2i-1}$ for i =1, 2, 3, ... , 7.

The 9-vector l contains the lengths of the PM fibers and the path lengths of the IOCs. The value of l_4 varies from 0 to *l(4)* meters with increments of *l(4)/(N-1)*, where the input N is the number of values for l_4. The value of l_7 varies from 0 to *l(7)* meters with increments of *l(7)/(M-1)*, where the input M is the number of values for l_7. The output of **StepIntens** is an $N \times M$ array containing the output intensities for the different values of l_4 and l_7.

The 2-vector e contains the two polarization extinction ratios ε_1 and ε_2. The 4-vector a contains the four miss-alignment angles α_1, α_2, α_3, and α_4 in radians. The 2-vector b contains the miss-alignment angles β_1, β_2 in radians. The 4-vector t has the magnitudes of the entries of the coil matrix $t = [t_{11}\ t_{12}\ t_{21}\ t_{22}]$, the 4-vector *tau* has the four phase shifts due to the coil; $tau = [\tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}]$. The input *phi* is the Sagnac phase, the input *input* is a 2-column unit vector defining the polarization strengths of the two orthogonal components of the electromagnetic field input, *w1* is the half width of the step function g and the integers N and M define the size of the output $N \times M$ matrix I.

With these inputs, the function **StepIntens** defines the 22-vector *phase* described earlier, it computes the six cosines and sines of the six rotations: *c1, c2, c3, c4, c5, c6,* and *s1, s2, s3, s4, s5, c6,* calls the four functions **Coefij_arr** (ij = 11, 12, 21, 22) and the four functions **Ph_arrCoij** (ij = 11, 12, 21, 22) described above, and computes the coefficients $2|a_1|^2(1+\cos\phi)$, $2|a_2|^2(1+\cos\phi)$, $2\cos\phi$, and $2|a_1\bar{a}_2|(1+\cos\phi)$ involved in Equation 3.10, where we have assumed that the phase $\alpha = 0$ (e.g. a_1 and a_2 real).

Next, there is a double loop where the output intensity is computed for the various values of the lengths l_4 and l_7. In this loop, the four 256-row vectors *B11, B12, B21,*

B22, are computed as described earlier. The differences $[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1$ are computed for all *s* and *p* and stored in the 256×256 matrix *Bij_uv* with the statement

Bij_uv = *Bij'* * ones(1,256) − ones(256,1) * *Buv*,

so that *Bij_uv* (*s*, *p*) = *Bij* (*s*) − *Buv* (*p*) = $[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1$.

The sums

$$\sum_{s=1}^{256}\sum_{p=1}^{256} C_{ij}(s)C_{uv}(p)\frac{\sin[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1}{[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1} \tag{A2}$$

are computed by a call to the function **StpSuvij** described next. These results are added up according to Equation 3.12 to give the output intensity when *g* is a step function.

The function **StpSuvij** computes the Sums A2. The inputs to this function are two arrays of coefficients *Cij* and *Cuv* and a 256×256 array of differences *Bij_uv*. The first step is to find the indices (*s*, *p*) such that

$$[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1 = 0, \tag{A3}$$

since for these indices $\frac{\sin[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1}{[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1}=1$. This is accomplished with the first statement

[x, y] = find(*Bij_uv* = = 0).

These indices are used in a loop to add the products $C_{ij}(s)C_{uv}(p)$ for all (*s*, *p*) such that Equation A3 hold. The statement

[i, j] = find(*Bij_uv*);

gives all indices (s, p) for which Equation A3 does not hold. A second loop uses these indices to add the products $C_{ij}(s)C_{uv}(p)\dfrac{\sin[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1}{[\beta_{ij}(s)-\beta_{uv}(p)]\omega_1}$ to the previous sum.

The function **SagnacBias** computes the error in the Sagnac phase shift due to imperfect polarizers. It has two integer inputs N and M which have been described earlier. This function simply defines values for the inputs to the function **StepIntens** in order to compute $I_\varepsilon(-\pi/2)$ then defines values for the inputs to the function **StepIntens** to compute $I_o(-\pi/2)$ and computes the Sagnac phase shift error ϕ_ε given by Equation 3.13. It then plots the error for the various values of l_4 and l_7.

When the frequency distribution function g is a triangular function we use the Matlab function **PL_Intens** instead of **StepIntens**. The function **PL_Intens** is identical to **StepIntens** except that it calls the function **PL_Suvij** to compute the sums in Equation 3.18. Also, the phases in **PL_Intens** do not include the factor *w1* like they do in **StepIntens**. Thus, *Bij_uv* (*s, p*) = *Bij* (*s*) − *Buv* (*p*) = $\beta_{ij}(s)-\beta_{uv}(p)$, and *wBij_uv* (*s, p*) = *w1***Bij_uv* (*s, p*) = $[\,\beta_{ij}(s)-\beta_{uv}(p)\,]\,\omega_1$.

When the frequency distribution function g is a Gaussian function we use the Matlab function **GausIntens** instead of **StepIntens**. Again, this function is identical to **StepIntens** except that the computation of the sums in Equation 3.14 are taken care of by **GausIntens** with statements of the form

sum(sum(*Cijuv*.*exp(-(*Bij*'*ones(1,256)-ones(256,1)**Buv*).^2))).

The phases in **GausIntens** do not include the factor *w1*, instead they include the factor (*sigma*/sqrt(8)). Thus, *Bij_uv* (*s, p*) = *Bij* (*s*) − *Buv* (*p*) =(*sigma*/sqrt(8))*$[\,\beta_{ij}(s)-\beta_{uv}(p)\,]$ as required in Equation 3.14.

PROGRAM LISTINGS

```
function [Err,Bias,I,Io] = SagnacBias(N,M)

%N=10;
%M=10;

% Del=[vector of length 7; index of refraction differences; "Deltas"]; Del_1 = (n_2 - n_1).
% l=[wavegide lenghts 1-7]=ones(1,7);
% e=[polarization extinction epsilons 1-2]=0.01*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% tau=[phase shifts due to coil, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a unit 2-column vector=[cos sin]';

z=9; % meters. 12m, 15m.

%Del=.00065*[0 1 .084/.00065 1 1 1 1];
Del=.00065*[0 1 .005/.00065 1 1 1 1];

%l=[.05 0 .05 15 2*z z 15]; % meters
l=[0 .2 .035 6 2*z z 6]; % meters; l(4) and l(7) will vary from -l(4) to l(4) and -l(7) to
                         % l(7), respectively.
e=[1 .001];
a=[.178 -.032 .032 -.032]; % radians
b=[.032 -.032]; % radians
t=1/sqrt(2)*[1 1 -1 1];
tau=1*[1 1 1 1];
phi=-pi/4;;
input=(1/sqrt(2))*[1 1]';
%input=[1 0]';
%sigma=1.67e+13; % 1/sec
sigma=1.654e+13; % 1/sec    lambda_o=1.3e-6 meters, sigma_lambda=15e-9 meters
w1=2*sigma;
%w1=2*sigma/10;

increm_i=1/(N-1);
increm_j=1/(M-1);
i=[1:N];
j=[1:M];
y=l(4)*(i-1)*increm_i;   % 0 <= r <= 1.
x=l(7)*(j-1)*increm_j;   % 0 <= s <= 1.

tic
```

```
 I=GausIntens(Del,l,e,a,b,t,tau,phi,input,w1,sigma,N,M);
% I=StepIntens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);
% I=PL_Intens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);

e=[1 0];
a=[0 0 .032 -.032];
b=[.032 -.032]; % radians;

 Io=GausIntens(Del,l,e,a,b,t,tau,phi,input,w1,sigma,N,M);
% Io=StepIntens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);
% Io=PL_Intens(Del,l,e,a,b,t,tau,phi,input,w1,N,M);

%save SagnacBias_10_15_3 I Io;     % Io_Step has epsilon1=1 and epsilon2=0.
                                   % Positive quadrant. w1=*/100, 50x50, (100x100=65.22 hours).
%save SagnacBias_10_23_3 I Io;     % Positive quadrant. w1=*/100, 100x100, (50x50=16.4
hours).
toc

Err=I-Io;
Bias=asin(Err./Io);

figure(1);
imagesc(x,y,I); axis image;
figure(2);
imagesc(x,y,Io); axis image;
figure(3);
imagesc(x,y,Err); axis image;
figure(4);
imagesc(x,y,Bias); axis image;
```

```
function I = GausIntens(Del,l,e,a,b,t,tau,phi,input,w1,sigma,N,M)

% This is for Gaussian Function g.

% N=60;
% M=60;
% Del=[vector of length 7; index of refraction differences; "Deltas"];
% e.g. Del_1 = (n_2 - n_1).
% l=[wavegide lenghts 1-9]=ones(1,9);
% e=[polarization extinction epsilons 1-2]=0.01*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% tau=[phase shifts due to coil T, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a 2-column vector=[1 1]';
```

```
c=3.e+8; %m/sec
input=input/sqrt(input(1)^2+input(2)^2);

phase=zeros(22,1);
for i=1:7
   phase(2*i)=Del(i)*l(i)/c;
end
phase(15)=tau(1)/( sigma/sqrt(8) );
phase(16)=tau(2)/( sigma/sqrt(8) );
phase(17)=tau(3)/( sigma/sqrt(8) );
phase(18)=tau(4)/( sigma/sqrt(8) );

phase(19)=phase(5);  % 0
phase(20)=phase(6);  % Del(3)*l(8)/c;
phase(21)=phase(1);  % 0
phase(22)=phase(2);  % Del(1)*l(9)/c;

phase=( sigma/sqrt(8) )*phase; % (sigma/sqrt(8))*phase is only for Gaussian g.
                        % Phases 8 and 14 will vary.
A1=0; % or pi/2 ( 0 or 90 degrees).
A2=0; % or pi/2 ( 0 or 90 degrees).

c1=cos(a(1));
s1=sin(a(1));

c2=cos(a(2));
s2=sin(a(2));

c3=cos(A1+a(3));
s3=sin(A1+a(3));

c4=cos(pi/4+b(1));
s4=sin(pi/4+b(1));

c5=cos(pi/4+b(2));
s5=sin(pi/4+b(2));

c6=cos(A2+a(4));
s6=sin(A2+a(4));

C11=Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C12=Coef12_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C21=Coef21_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C22=Coef22_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);

C1111=C11*C11';
C2222=C22*C22';
```

```
C1212=C12*C12';
C2121=C21*C21';
C1221=C12*C21';
C1112=C11*C12';
C1121=C11*C21';
C2122=C21*C22';
C2212=C22*C12';

L11=Ph_arrCo11';
L12=Ph_arrCo12';
L21=Ph_arrCo21';
L22=Ph_arrCo22';

Err=zeros(N,M);
Rel_Err=Err;
I=Err;

a1a1=input(1)^2;
a2a2=input(2)^2;
cosphi2=2*cos(phi);
cosphi_2=2+2*cos(phi);
a1a1cosphi_2=a1a1*cosphi_2;
a2a2cosphi_2=a2a2*cosphi_2;
a1a2cosphi_2=abs(input(1)*input(2))*cosphi_2;

increm_i=1/(N-1);
increm_j=1/(M-1);

for i=1:N
   for j=1:M
      r=(i-1)*increm_i;   % 0 <= r <= 1.
      s=(j-1)*increm_j;   % 0 <= s <= 1.

      phase(8)=( ( sigma/sqrt(8) )/c )*l(4)*Del(4)*(2*r-1);    % To go from -l(4) to l(4).
      phase(14)=( ( sigma/sqrt(8) )/c )*l(7)*Del(7)*(2*s-1);   % To go from -l(7) to l(7).

      %phase(8)= ( ( sigma/sqrt(8) )/c )*(l(4)/2)*Del(4)*(1+r);         % To go from l(4)/2 to l(4).
      %phase(14)= ( ( sigma/sqrt(8) )/c )*(l(7)/2)*Del(7)*(1+s);        % To go from l(7)/2 to l(7).

      Phase_M=phase*ones(1,256);

      B11=sum(Phase_M(L11));
      B12=sum(Phase_M(L12));
      B21=sum(Phase_M(L21));
      B22=sum(Phase_M(L22));

      P1=a1a1cosphi_2*...
         sum(sum(C1111.*exp(-(B11'*ones(1,256)-ones(256,1)*B11).^2)));
```

```
      P2=a2a2cosphi_2*...
         sum(sum(C2222.*exp(-(B22'*ones(1,256)-ones(256,1)*B22).^2)));
      P3=sum(sum(C1212.*exp(-(B12'*ones(1,256)-ones(256,1)*B12).^2)));
      P4=sum(sum(C2121.*exp(-(B21'*ones(1,256)-ones(256,1)*B21).^2)));
      P5=cosphi2*...
         sum(sum(C1221.*exp(-(B12'*ones(1,256)-ones(256,1)*B21).^2)));
      P6=a1a2cosphi_2*...
         sum(sum(C1112.*exp(-(B11'*ones(1,256)-ones(256,1)*B12).^2)));
      P7=a1a2cosphi_2*...
         sum(sum(C1121.*exp(-(B11'*ones(1,256)-ones(256,1)*B21).^2)));
      P8=a1a2cosphi_2*...
         sum(sum(C2122.*exp(-(B21'*ones(1,256)-ones(256,1)*B22).^2)));
      P9=a1a2cosphi_2*...
         sum(sum(C2212.*exp(-(B22'*ones(1,256)-ones(256,1)*B12).^2)));

      I(i,j)=P1+P2+P3+P4+P5+P6+P7+P8+P9;

   end
end

I=( sqrt(2*pi)/sigma )*I;
```

```
function I = StepIntens(Del,l,e,a,b,t,tau,phi,input,w1,N,M)

% This is for a Step Function g.

% N=60;
% M=60;
% Del=[vector of length 7; index of refraction differences; "Deltas"];
% e.g. Del_1 = (n_2 - n_1).
% l=[wavegide lenghts 1-9]=ones(1,9);
% e=[polarization extinction epsilons 1 and 2]=0.001*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% tau=[phase shifts due to coil, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a 2-column unit vector=[cos sin]';

c=3.e+8; % m/sec
input=input/sqrt(input(1)^2+input(2)^2);

phase=zeros(22,1);
for i=1:7
   phase(2*i)=( w1/c )*Del(i)*l(i);  % w1*phase is only for Step Function g.
end
phase(15)=tau(1);
```

```
phase(16)=tau(2);
phase(17)=tau(3);
phase(18)=tau(4);

phase(19)=phase(5);  % 0
phase(20)=phase(6);  % ( w1/c )*Del(3)*l(8);
phase(21)=phase(1);  % 0
phase(22)=phase(2);  % ( w1/c )*Del(1)*l(9);

% phases 8 and 14 will vary.

A1=0; % or pi/2 ( 0 or 90 degrees).
A2=0; % or pi/2 ( 0 or 90 degrees).

c1=cos(a(1));
s1=sin(a(1));

c2=cos(a(2));
s2=sin(a(2));

c3=cos(A1+a(3));
s3=sin(A1+a(3));

c4=cos(pi/4+b(1));
s4=sin(pi/4+b(1));

c5=cos(pi/4+b(2));
s5=sin(pi/4+b(2));

c6=cos(A2+a(4));
s6=sin(A2+a(4));

C11=Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C12=Coef12_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C21=Coef21_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C22=Coef22_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);

C1111=C11*C11';
C2222=C22*C22';
C1212=C12*C12';
C2121=C21*C21';
C1221=C12*C21';
C1112=C11*C12';
C1121=C11*C21';
C2122=C21*C22';
C2212=C22*C12';

L11=Ph_arrCo11';
```

```
L12=Ph_arrCo12';
L21=Ph_arrCo21';
L22=Ph_arrCo22';

Err=zeros(N,M);
Rel_Err=Err;
I=Err;

a1a1=input(1)^2;
a2a2=input(2)^2;
cosphi2=2*cos(phi);
cosphi_2=2+2*cos(phi);
a1a1cosphi_2=a1a1*cosphi_2;
a2a2cosphi_2=a2a2*cosphi_2;
a1a2cosphi_2=abs(input(1)*input(2))*cosphi_2;

increm_i=1/(N-1);
increm_j=1/(M-1);

for i=1:N
   for j=1:M
      r=(i-1)*increm_i;   % 0 <= r <= 1.
      s=(j-1)*increm_j;   % 0 <= s <= 1.

      %phase(8)=w1*l(4)*(2*r-1)*Del(4)/c;    % To go from -l(4) to l(4).
      %phase(14)=w1*l(7)*(2*s-1)*Del(7)/c;   % To go from -l(7) to l(7).

      %phase(8)=w1*l(4)*(r-1)*Del(4)/c;      % To go from -l(4) to 0.
      %phase(14)=w1*l(7)*(s-1)*Del(7)/c;     % To go from -l(7) to 0.

      phase(8)=(w1/c)*Del(4)*l(4)*r;         % To go from 0 to l(4).
      phase(14)=(w1/c)*Del(7)*l(7)*s;        % To go from 0 to l(7).

      %phase(8)= (l(4)/2)*(1+r)*Del(4)/c;    % To go from l(4)/2 to l(4).
      %phase(14)= (l(7)/2)*(1+s)*Del(7)/c;   % To go from l(7)/2 to l(7).

      Phase_M=phase*ones(1,256);

      B11=sum(Phase_M(L11));
      B12=sum(Phase_M(L12));
      B21=sum(Phase_M(L21));
      B22=sum(Phase_M(L22));
      %BA=sum(Phase_M(LA));

      B11_11=B11'*ones(1,256)-ones(256,1)*B11;
      P1=a1a1cosphi_2*StpSuvij(C1111,B11_11);

      B22_22=B22'*ones(1,256)-ones(256,1)*B22;
```

```
        P2=a2a2cosphi_2*StpSuvij(C2222,B22_22);

        B12_12=B12'*ones(1,256)-ones(256,1)*B12;
        P3=StpSuvij(C1212,B12_12);

        B21_21=B21'*ones(1,256)-ones(256,1)*B21;
        P4=StpSuvij(C2121,B21_21);

        B12_21=B12'*ones(1,256)-ones(256,1)*B21;
        P5=cosphi2*StpSuvij(C1221,B12_21);

        B11_12=B11'*ones(1,256)-ones(256,1)*B12;
        P6=a1a2cosphi_2*StpSuvij(C1112,B11_12);

        B11_21=B11'*ones(1,256)-ones(256,1)*B21;
        P7=a1a2cosphi_2*StpSuvij(C1121,B11_21);

        B21_22=B21'*ones(1,256)-ones(256,1)*B22;
        P8=a1a2cosphi_2*StpSuvij(C2122,B21_22);

        B22_12=B22'*ones(1,256)-ones(256,1)*B12;
        P9=a1a2cosphi_2*StpSuvij(C2212,B22_12);

        %sum(sum(Cij*Cuv'.*exp(-(Bij*ones(1,256)-ones(256,1)*Buv').^2)));

        I(i,j)=P1+P2+P3+P4+P5+P6+P7+P8+P9;

    end
end

I=(pi/(2*w1))*I;
```

```
function SS = StpSuvij(Cijuv,Buv_ij)

% Buv_ij may be Buv-Bij or Buv+Bij. Buv_ij has been multiplied by w1 already.

SS=0;

[x,y]=find(Buv_ij == 0);

for s=1:size(x)
    SS=SS+Cijuv(x(s),y(s));
end

[i,j]=find(Buv_ij);

for s=1:size(i)
```

```
   SS=SS+Cijuv(i(s),j(s))*( sin( Buv_ij(i(s),j(s))) / Buv_ij(i(s),j(s)) );
end
```

```
function I=PL_Intens(Del,l,e,a,b,t,tau,phi,input,w1,N,M)

% This is for Triangular Function g.

%N=60;
%M=60;

% Del=[vector of length 7; index of refraction differences; "Deltas"];
% e.g. Del_1 = (n_2 - n_1).
% l=[wavegide lenghts 1-9]=ones(1,9);
% e=[polarization extinction epsilons 1-2]=0.01*[1 1];
% a=[coupling misalignment small angles 1-4]=(pi/100)*[1 1 1 1];
% b=[coupling misalignment small angles 1-2]=(pi/100)*[1 1];
% t=magnitud of entries in the coil matrix, t=[1 1 1 1];
% tau=[phase shifts due to coil T, 1-4]=[1 1 1 1];
% phi=Sagnac phase
% input = a 2-column vector=[1 1]';

c=3.e+8; %m/sec
input=input/sqrt(input(1)^2+input(2)^2);

phase=zeros(22,1);
for i=1:7
   phase(2*i)=Del(i)*l(i)/c;
end
phase(15)=tau(1)/w1;  % tau/w1 is only for PL function g.
phase(16)=tau(2)/w1;
phase(17)=tau(3)/w1;
phase(18)=tau(4)/w1;

phase(19)=phase(5);  % 0
phase(20)=phase(6);  % Del(3)*l(8)/c;
phase(21)=phase(1);  % 0
phase(22)=phase(2);  % Del(1)*l(9)/c;

% phases 8 and 14 will vary.

A1=0; % or pi/2 ( 0 or 90 degrees).
A2=0; % or pi/2 ( 0 or 90 degrees).

c1=cos(a(1));
s1=sin(a(1));

c2=cos(a(2));
```

```
s2=sin(a(2));

c3=cos(A1+a(3));
s3=sin(A1+a(3));

c4=cos(pi/4+b(1));
s4=sin(pi/4+b(1));

c5=cos(pi/4+b(2));
s5=sin(pi/4+b(2));

c6=cos(A2+a(4));
s6=sin(A2+a(4));

C11=Coef11_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C12=Coef12_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C21=Coef21_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);
C22=Coef22_arr(s1,s2,s3,s4,s5,s6,c1,c2,c3,c4,c5,c6,e(1),e(2),t);

C1111=C11*C11';
C2222=C22*C22';
C1212=C12*C12';
C2121=C21*C21';
C1221=C12*C21';
C1112=C11*C12';
C1121=C11*C21';
C2122=C21*C22';
C2212=C22*C12';

L11=Ph_arrCo11';
L12=Ph_arrCo12';
L21=Ph_arrCo21';
L22=Ph_arrCo22';

Err=zeros(N,M);
Rel_Err=Err;
I=Err;

a1a1=input(1)^2;
a2a2=input(2)^2;
cosphi2=2*cos(phi);
cosphi_2=2+2*cos(phi);
a1a1cosphi_2=a1a1*cosphi_2;
a2a2cosphi_2=a2a2*cosphi_2;
a1a2cosphi_2=abs(input(1)*input(2))*cosphi_2;

increm_i=1/(N-1);
increm_j=1/(M-1);
```

```
for i=1:N
   for j=1:M
      r=(i-1)*increm_i;   % 0 <= r <= 1.
      s=(j-1)*increm_j;   % 0 <= s <= 1.

      phase(8)=l(4)*(2*r-1)*Del(4)/c;    % To go from -l(4) to l(4).
      phase(14)=l(7)*(2*s-1)*Del(7)/c;   % To go from -l(7) to l(7).

      %phase(8)= (l(4)/2)*(1+r)*Del(4)/c;          % To go from l(4)/2 to l(4).
      %phase(14)= (l(7)/2)*(1+s)*Del(7)/c;         % To go from l(7)/2 to l(7).

      Phase_M=phase*ones(1,256);

      B11=sum(Phase_M(L11));
      B12=sum(Phase_M(L12));
      B21=sum(Phase_M(L21));
      B22=sum(Phase_M(L22));

      B11_11=B11'*ones(1,256)-ones(256,1)*B11;
      wB11_11=w1*B11_11;
      P1=a1a1cosphi_2*PL_Suvij(w1,C1111,B11_11,wB11_11);

      B22_22=B22'*ones(1,256)-ones(256,1)*B22;
      wB22_22=w1*B22_22;
      P2=a2a2cosphi_2*PL_Suvij(w1,C2222,B22_22,wB22_22);

      B12_12=B12'*ones(1,256)-ones(256,1)*B12;
      wB12_12=w1*B12_12;
      P3=PL_Suvij(w1,C1212,B12_12,wB12_12);

      B21_21=B21'*ones(1,256)-ones(256,1)*B21;
      wB21_21=w1*B21_21;
      P4=PL_Suvij(w1,C2121,B21_21,wB21_21);

      B12_21=B12'*ones(1,256)-ones(256,1)*B21;
      wB12_21=w1*B12_21;
      P5=cosphi2*PL_Suvij(w1,C1221,B12_21,wB12_21);

      B11_12=B11'*ones(1,256)-ones(256,1)*B12;
      wB11_12=w1*B11_12;
      P6=a1a2cosphi_2*PL_Suvij(w1,C1112,B11_12,wB11_12);

      B11_21=B11'*ones(1,256)-ones(256,1)*B21;
      wB11_21=w1*B11_21;
      P7=a1a2cosphi_2*PL_Suvij(w1,C1121,B11_21,wB11_21);

      B21_22=B21'*ones(1,256)-ones(256,1)*B22;
```

```
    wB21_22=w1*B21_22;
    P8=a1a2cosphi_2*PL_Suvij(w1,C2122,B21_22,wB21_22);

    B22_12=B22'*ones(1,256)-ones(256,1)*B12;
    wB22_12=w1*B22_12;
    P9=a1a2cosphi_2*PL_Suvij(w1,C2212,B22_12,wB22_12);

    I(i,j)=P1+P2+P3+P4+P5+P6+P7+P8+P9;

  end
end

I=2*pi*I/w1^2;
```

```
function SS = PL_Suvij(w1,Cijuv,Buv_ij,wBuv_ij)

w1_x23=2*(w1/3);
w1_4=4/w1;
SS=0;

[x,y]=find(Buv_ij == 0);

for s=1:size(x)
  SS=SS+w1_x23*Cijuv(x(s),y(s));
end

[i,j]=find(Buv_ij);

for s=1:size(i)
  SS=SS+w1_4*(Cijuv(i(s),j(s))/Buv_ij(i(s),j(s))^2)*...
     (1-sin(wBuv_ij(i(s),j(s)))/(wBuv_ij(i(s),j(s))));
end
```

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